## CHAPTER 10

**P. E. 10.1** (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\times 10^8} = \underline{31.42 \text{ ns}},$$

$$\lambda = uT = 3\times 10^8 \times 31.42\times 10^{-9} = \underline{9.425 \text{ m}}$$

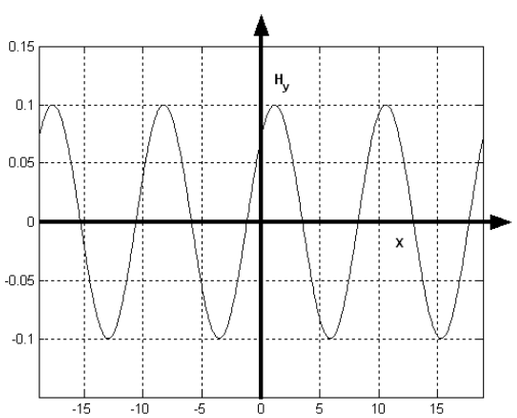
$$k = \beta = 2\pi / \lambda = \underline{0.6667 \text{ rad/m}}$$

(b) $t_1 = T/8 = \underline{3.927 \text{ ns}}$

(c)

$$\boldsymbol{H}(t = t_1) = 0.1\cos(2\times 10^8 \frac{\pi}{8\times 10^8} - 2x/3)\boldsymbol{a}_y = 0.1\cos(2x/3 - \pi/4)\boldsymbol{a}_y$$

as sketched below.




**P. E. 10.2** Let $x_o = \sqrt{1 + (\sigma/\omega\varepsilon)^2}$, then

$$\alpha = \omega\sqrt{\frac{\mu_o\varepsilon_o}{2}\mu_r\varepsilon_r(x_o - 1)} = \frac{\omega}{c}\sqrt{\frac{16}{2}}\sqrt{x_o - 1}$$

or $$\sqrt{x_o - 1} = \frac{\alpha c}{\omega\sqrt{8}} = \frac{1/3\times 3\times 10^8}{10^8\sqrt{8}} = \frac{1}{\sqrt{8}} \longrightarrow x_o = 9/8$$

$$x_o^2 = \frac{81}{64} = 1 + (\sigma/\omega\varepsilon)^2 \longrightarrow \frac{\sigma}{\omega\varepsilon} = 0.5154$$

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$\tan 2\theta_\eta = 0.5154 \longrightarrow \theta_\eta = 13.63^o$

$$\frac{\beta}{\alpha} = \sqrt{\frac{x_o + 1}{x_o - 1}} = \sqrt{17}$$

(a) $\beta = \alpha\sqrt{17} = \frac{\sqrt{17}}{3} =$ 1.374 rad/m

(b) $\frac{\sigma}{\omega\varepsilon} = 0.5154$

(c) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x_o}} = \frac{120\pi\sqrt{2/8}}{\sqrt{9/8}} = 177.72$

$\eta = 177.72\angle 13.63^o\ \Omega$

(d) $u = \frac{\omega}{\beta} = \frac{10^8}{1.374} = 7.278\times 10^7$ m/s

(e) $\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E \longrightarrow \boldsymbol{a}_z \times \boldsymbol{a}_x = \boldsymbol{a}_H \longrightarrow \boldsymbol{a}_H = \boldsymbol{a}_y$

$$\boldsymbol{H} = \frac{0.5}{177.5} e^{-z/3} \sin(10^8 t - \beta z - 13.63^o)\boldsymbol{a}_y = 2.817 e^{-z/3} \sin(10^8 t - \beta z - 13.63^o)\boldsymbol{a}_y \text{ mA/m}$$

**P. E. 10.3** (a) Along -z direction

(b) $\lambda = \frac{2\pi}{\beta} = 2\pi/2 =$ 3.142 m

$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} =$ 15.92 MHz

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{(1)\varepsilon_r}$$

or $\sqrt{\varepsilon_r} = \beta c/\omega = \frac{3\times 10^8 \times 2}{10^8} = 6 \longrightarrow \varepsilon_r = 36$

(c) $\theta_\eta = 0, |\eta| = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{1/\varepsilon_r} = \frac{120\pi}{6} = 20\pi$

$\boldsymbol{a}_k = \boldsymbol{a}_E \times \boldsymbol{a}_H \longrightarrow -\boldsymbol{a}_z = \boldsymbol{a}_y \times \boldsymbol{a}_H \longrightarrow \boldsymbol{a}_H = \boldsymbol{a}_x$

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$$\boldsymbol{H} = \frac{50}{20\pi}\sin(\omega t + \beta z)\boldsymbol{a}_x = \underline{\underline{795.8\sin(10^8 t + 2z)\boldsymbol{a}_x}} \text{ mA/m}$$

**P. E. 10.4** (a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{10^9\pi \times 4 \times \frac{10^{-9}}{36\pi}} = 0.09$$

$$\alpha \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 - 1\right]} = \frac{\omega}{2c}\sqrt{\mu_r\varepsilon_r}\,\frac{\sigma}{\omega\varepsilon} = \frac{10^9\pi}{2\times 3\times 10^8}(2)(0.09) = 0.9425 \text{ Np/m}$$

$$\beta \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1\right]} = \frac{10^9\pi}{3\times 10^8}\sqrt{2[2 + 0.5(0.09)^2]} = 20.965 \text{ rad/m}$$

$$\boldsymbol{E} = 30e^{-0.9425y}\cos(10^9\pi t - 20.96y + \pi/4)\boldsymbol{a}_z$$

At t = 2ns, y = 1m,

$$\boldsymbol{E} = 30e^{-0.9425}\cos(2\pi - 20.96 + \pi/4)\boldsymbol{a}_z = \underline{\underline{2.844\boldsymbol{a}_z}} \text{ V/m}$$

(b) $\beta y = 10^o = \frac{10\pi}{180}$ rad

or

$$y = \frac{\pi}{18}\frac{1}{\beta} = \frac{\pi}{18\times 20.965} = \underline{\underline{8.325 \text{ mm}}}$$

(c) $30(0.6) = 30\ e^{-\alpha y}$

$$y = \frac{1}{\alpha}\ln(1/0.6) = \frac{1}{0.9425}\ln\frac{1}{0.6} = \underline{\underline{542 \text{ mm}}}$$

(d)

$$|\eta| \cong \frac{\sqrt{\mu/\varepsilon}}{[1 + \frac{1}{4}(0.09)^2]} = \frac{60\pi}{1.002} = 188.11\ \Omega$$

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$2\theta_\eta = \tan^{-1} 0.09 \longrightarrow \theta_\eta = 2.571^o$

$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_y \times \boldsymbol{a}_z = \boldsymbol{a}_x$

$$\boldsymbol{H} = \frac{30}{188.11} e^{-0.9425y} \cos(10^9 \pi t - 20.96y + \pi/4 - 2.571^o)\boldsymbol{a}_x$$

At y = 2m, t = 2ns,

$\boldsymbol{H} = (0.1595)(0.1518)\ \cos(-34.8963 rad)\boldsymbol{a}_x = \underline{\underline{-22.83\boldsymbol{a}_x}}$ mA/m

**P. E. 10.5**

$$I_s = \int_0^w \int_0^\infty J_{xs} dy dz = J_{xs}(0) \int_0^w dy \int_0^\infty e^{-z(1+j)/\delta} dz = \frac{J_{xs}(0) w\delta}{1+j}$$

$$\underline{\underline{|I_s| = \frac{J_{xs}(0) w\delta}{\sqrt{2}}}}$$

**P. E. 10.6** (a)

$$\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu \sigma} = \frac{1.3\times10^{-3}}{2}\sqrt{\pi \times 10^7 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7} = \underline{\underline{24.16}}$$

(b)

$$\frac{R_{ac}}{R_{dc}} = \frac{1.3\times10^{-3}}{2}\sqrt{\pi \times 2\times10^9 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7} = \underline{\underline{341.7}}$$

**P. E. 10.7**

$$E = \text{Re}[\boldsymbol{E}_s e^{j\omega t}] = \text{Re}\left[E_o e^{j\omega t} e^{-j\beta z}\boldsymbol{a}_x + E_o e^{-j\pi/2} e^{j\omega t} e^{-j\beta z}\boldsymbol{a}_y\right]$$

$$= E_o \cos(\omega t - \beta z)\boldsymbol{a}_x + E_o \cos(\omega t - \beta z - \pi/2)\boldsymbol{a}_y$$

$$= E_o \cos(\omega t - \beta z)\boldsymbol{a}_x + E_o \sin(\omega t - \beta z)\boldsymbol{a}_y$$

At z = 0, $E_x = E_o \cos\omega t, E_y = E_o \sin\omega t$

$$\cos^2\omega t + \sin^2\omega t = 1 \longrightarrow \left(\frac{E_x}{E_o}\right)^2 + \left(\frac{E_y}{E_o}\right)^2 = 1$$

which describes a circle. Hence the polarization is circular.

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**P. E. 10.8**

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\eta H_o^{\,2}\boldsymbol{a}_x$$

(a) Let $f(x,z) = x + y - 1 = 0$

$$\boldsymbol{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\boldsymbol{a}_x + \boldsymbol{a}_y}{\sqrt{2}}, \qquad d\boldsymbol{S} = \text{dS}\boldsymbol{a}_{\text{n}}$$

$$P_t = \int \mathbf{P}.\text{d}\mathbf{S} = \mathbf{P}.S\mathbf{a}_{\mathbf{n}} = \frac{1}{2}\eta H_o^{\,2}\boldsymbol{a}_x.\frac{\boldsymbol{a}_x + \boldsymbol{a}_y}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}(120\pi)(0.2)^2(0.1)^2 = \underline{\underline{53.31 \text{ mW}}}$$

(b) $d\boldsymbol{S} = \text{dydz}\boldsymbol{a}_{\text{x}}$, $P_t = \int \mathcal{P}.\text{d}\mathbf{S} = \frac{1}{2}\eta H_o^{\,2}S$

$$P_t = \frac{1}{2}(120\pi)(0.2)^2\pi(0.05)^2 = \underline{\underline{59.22 \text{ mW}}}$$

**P. E. 10.9** $\eta_1 = \eta_o = 120\pi, \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{2}$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 2/3, \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3$$

$$E_{ro} = \Gamma E_{io} = -\frac{10}{3}$$

$$\underline{\underline{\boldsymbol{E}_{rs} = -\frac{10}{3}e^{j\beta_1 z}\boldsymbol{a}_x \text{ V/m}}}$$

where $\beta_1 = \omega/c = 100\pi/3$.

$$E_{to} = \tau E_{io} = \frac{20}{3}$$

$$\underline{\underline{\boldsymbol{E}_{ts} = \frac{20}{3}e^{-j\beta_2 z}\boldsymbol{a}_x \text{ V/m}}}$$

where $\beta_2 = \omega\sqrt{\varepsilon_r}/c = 2\beta_1 = 200\pi/3$.

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**P. E. 10.10**

$$\alpha_1 = 0, \quad \beta_1 = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\omega}{c} = 5 \longrightarrow \omega = 5c/2 = 7.5\times 10^8$$

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{0.1}{7.5\times 10^8 \times 4 \times \frac{10^{-9}}{36\pi}} = 1.2\pi$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1+1.44\pi^2}-1\right]} = 6.021$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1+1.44\pi^2}+1\right]} = 7.826$$

$$|\eta_2| = \frac{60\pi}{\sqrt[4]{1+1.44\pi^2}} = 95.445, \eta_1 = 120\pi\sqrt{\varepsilon_{r1}} = 754$$

$$\tan 2\theta_{\eta_2} = 1.2\pi \longrightarrow \theta_{\eta_2} = 37.57^o$$

$$\eta_2 = 95.445\angle 37.57^o$$

(a)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.445\angle 37.57^o - 754}{95.445\angle 37.57^o + 754} = \underline{\underline{0.8186\angle 171.08^o}}$$

$$\tau = 1 + \Gamma = \underline{\underline{0.2295\angle 33.56^o}}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.8186}{1-0.8186} = \underline{\underline{10.025}}$$

(b) $\boldsymbol{E}_i = 50\sin(\omega t - 5x)\boldsymbol{a}_y = \text{Im}(\boldsymbol{E}_{is}e^{j\omega t})$, where $E_{is} = 50e^{-j5x}a_y$.

$$E_{ro} = \Gamma E_{io} = 0.8186e^{j171.08^o}(50) = 40.93e^{j171.08^o}$$

$$\boldsymbol{E}_{rs} = 40.93e^{j5x+j171.08^o}\boldsymbol{a}_y$$

$$\boldsymbol{E}_r = \text{Im}(\boldsymbol{E}_{rs}e^{j\omega t}) = \underline{\underline{40.93\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_y}} \text{ V/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = -\boldsymbol{a}_x \times \boldsymbol{a}_y = -\boldsymbol{a}_z$$

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$$\boldsymbol{H}_r = -\frac{40.93}{754}\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_z = \underline{\underline{-0.0543\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_z}} \text{ A/m}$$

(c)

$$E_{to} = \tau E_{io} = 0.229e^{j33.56^o}(50) = 11.475e^{j33.56^o}$$

$$\boldsymbol{E}_{ts} = 11.475e^{-j\beta_2 x + j33.56^o}e^{-\alpha_2 x}\boldsymbol{a}_y$$

$$\boldsymbol{E}_t = \text{Im}(\boldsymbol{E}_{ts}e^{j\omega t}) = \underline{\underline{11.475e^{-6.021x}\sin(\omega t - 7.826x + 33.56^o)\boldsymbol{a}_y}} \text{ V/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_x \times \boldsymbol{a}_y = \boldsymbol{a}_z$$

$$\boldsymbol{H}_t = \frac{11.495}{95.445}e^{-6.021x}\sin(\omega t - 7.826x + 33.56^o - 37.57^o)\boldsymbol{a}_z$$

$$= \underline{\underline{0.1202e^{-6.021x}\sin(\omega t - 7.826x - 4.01^o)\boldsymbol{a}_z}} \text{ A/m}$$

(d)

$$\mathcal{P}_{1\text{ave}} = \frac{E_{io}^{\ 2}}{2\eta_1}\boldsymbol{a}_x + \frac{E_{ro}^{\ 2}}{2\eta_1}(-\boldsymbol{a}_x) = \frac{1}{2(240\pi)}[50^2\boldsymbol{a}_x - 40.93^2\boldsymbol{a}_x] = \underline{\underline{0.5469\boldsymbol{a}_x}} \text{ W/m}^2$$

$$\boldsymbol{P}_{2\text{ave}} = \frac{E_{to}^{\ 2}}{2|\eta_2|}e^{-2\alpha_2 x}\cos\theta_{\eta_2}\boldsymbol{a}_x = \frac{(11.475)^2}{2(95.445)}\cos 37.57^o e^{-2(6.021)x}\boldsymbol{a}_x = \underline{\underline{0.5469e^{-12.04x}\boldsymbol{a}_x}} \text{ W/m}^2$$

**P. E. 10.11** (a)

$$\boldsymbol{k} = -2\boldsymbol{a}_y + 4\boldsymbol{a}_z \longrightarrow k = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\omega = kc = 3\times 10^8\sqrt{20} = \underline{\underline{1.342\times 10^9 \text{ rad/s}}},$$

$$\lambda = 2\pi / k = \underline{\underline{1.405\text{m}}}$$

$$\text{(b) } \boldsymbol{H} = \frac{\boldsymbol{a}_k \times \boldsymbol{E}}{\eta_o} = \frac{(-2\boldsymbol{a}_y + 4\boldsymbol{a}_z)}{\sqrt{20}(120\pi)} \times (10\boldsymbol{a}_y + 5\boldsymbol{a}_z)\cos(\omega t - \boldsymbol{k}.\boldsymbol{r})$$

$$= \underline{\underline{-29.66\cos(1.342x10^9 t + 2y - 4z)\boldsymbol{a}_x}} \text{ mA/m}$$

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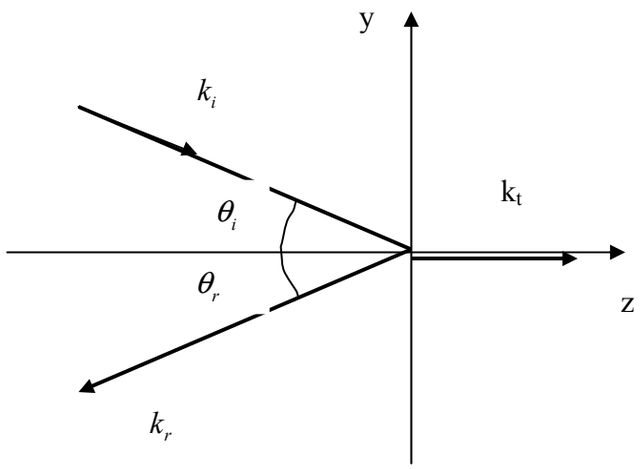
(c) $\mathcal{P}_{\text{ave}} = \dfrac{|E_o|^2}{2\eta_o}\boldsymbol{a}_k = \dfrac{125}{2(120\pi)}\dfrac{(-2\boldsymbol{a}_y + 4\boldsymbol{a}_z)}{\sqrt{20}} = \underline{\underline{-74.15\boldsymbol{a}_y + 148.9\boldsymbol{a}_z}}$ mW/m$^2$

**P. E. 10.12** (a)




$$\tan\theta_i = \frac{k_{iy}}{k_{iz}} = \frac{2}{4} \longrightarrow \underline{\underline{\theta_i = 26.56 = \theta_r}}$$

$$\sin\theta_t = \sqrt{\frac{\mu_1\,\varepsilon_1}{\mu_2\,\varepsilon_2}}\sin\theta_i = \frac{1}{2}\sin 26.56^o \longrightarrow \underline{\underline{\theta_t = 12.92^o}}$$

(b) $\eta_1 = \eta_o, \eta_2 = \eta_o/2$ **E** is parallel to the plane of incidence. Since $\mu_1 = \mu_2 = \mu_o$,

we may use the result of Prob. 10.42, i.e.

$$\Gamma_{\shortparallel} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\tan(-13.64^o)}{\tan(39.48^o)} = \underline{\underline{-0.2946}}$$

$$\tau_{\shortparallel} = \frac{2\cos 26.56^o \sin 12.92^o}{\sin 39.48^o \cos(-13.64^o)} = \underline{\underline{0.6474}}$$

(c) $\boldsymbol{k}_r = -\beta_1 \sin\theta_r \boldsymbol{a}_y - \beta_1 \cos\theta_r \boldsymbol{a}_z$. Once $k_r$ is known, $E_r$ is chosen such that

$\boldsymbol{k}_r . \boldsymbol{E}_r = 0$ or $\nabla . \boldsymbol{E}_r = 0$. Let

$$\boldsymbol{E}_r = \pm E_{or}(-\cos\theta_r \boldsymbol{a}_y + \sin\theta_r \boldsymbol{a}_z)\cos(\omega t + \beta_1 \sin\theta_r y + \beta_1 \cos\theta_r z)$$

Only the positive sign will satisfy the boundary conditions. It is evident that

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$$\boldsymbol{E}_i = E_{oi}(\cos\theta_i \boldsymbol{a}_y + \sin\theta_i \boldsymbol{a}_z)\cos(\omega t + 2y - 4z)$$

Since $\theta_r = \theta_i$,

$$E_{or}\cos\theta_r = \Gamma_{//} E_{oi}\cos\theta_i = 10\Gamma_{//} = -2.946$$

$$E_{or}\sin\theta_r = \Gamma_{//} E_{oi}\sin\theta_i = 5\Gamma_{//} = -1.473$$

$$\beta_1 \sin\theta_r = 2, \beta_1 \cos\theta_r = 4$$

i.e.

$$E_r = -(2.946a_y - 1.473a_z)\cos(\omega t + 2y + 4z)$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{(10\boldsymbol{a}_y + 5\boldsymbol{a}_z)\cos(\omega t + 2y - 4z) + (-2.946\boldsymbol{a}_y + 1.473\boldsymbol{a}_z)\cos(\omega t + 2y + 4z)}}$$

V/m

(d) $\boldsymbol{k}_t = -\beta_2 \sin\theta_t \boldsymbol{a}_y + \beta_2 \cos\theta_t \boldsymbol{a}_z$. Since $k_r \bullet E_r = 0$, let

$$\boldsymbol{E}_t = E_{ot}(\cos\theta_t \boldsymbol{a}_y + \sin\theta_t \boldsymbol{a}_z)\cos(\omega t + \beta_2 y \sin\theta_t - \beta_2 z\cos\theta_t)$$

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \beta_1\sqrt{\varepsilon_{r2}} = 2\sqrt{20}$$

$$\sin\theta_t = \frac{1}{2}\sin\theta_i = \frac{1}{2\sqrt{5}}, \qquad \cos\theta_t = \frac{\sqrt{9}}{\sqrt{20}}$$

$$\beta_2\cos\theta_t = 2\sqrt{20}\sqrt{\frac{19}{20}} = 8.718$$

$$E_{ot}\cos\theta_t = \tau_{//} E_{oi}\cos\theta_t = 0.6474\sqrt{125}\sqrt{\frac{19}{20}} = 7.055$$

$$E_{ot}\sin\theta_t = \tau_{//} E_{oi}\sin\theta_t = 0.6474\sqrt{125}\sqrt{\frac{1}{20}} = 1.6185$$

Hence

$$\boldsymbol{E}_2 = \boldsymbol{E}_t = \underline{\underline{(7.055\boldsymbol{a}_y + 1.6185\boldsymbol{a}_z)\cos(\omega t + 2y - 8.718z)}} \text{ V/m}$$

(d) $\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 2 \longrightarrow \underline{\underline{\theta_{B//} = 63.43^o}}$

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**P.E. 10.13**

$$S_i = \frac{1+0.4}{1-0.4} = \frac{1.4}{0.6} = \underline{2.333}$$

$$S_o = \frac{1+0.2}{1-0.2} = \frac{1.2}{0.8} = \underline{1.5}$$

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**Prob. 10.1** (a) Wave propagates along $+\mathbf{a}_x$.

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times 10^6} = \underline{1\mu s}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{1.047\text{m}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{6} = \underline{1.047 \times 10^6 \text{m}/s}$$

(c) At $t$=0, $E_z = 25\sin(-6x) = -25\sin 6x$

At $t=T/8$, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{8} - 6x) = 25\sin(\frac{\pi}{4} - 6x)$

At $t=T/4$, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{4} - 6x) = 25\sin(-6x + 90^o) = 25\cos 6x$

At $t=T/2$, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{2} - 6x) = 25\sin(-6x + \pi) = 25\sin 6x$

These are sketched below.

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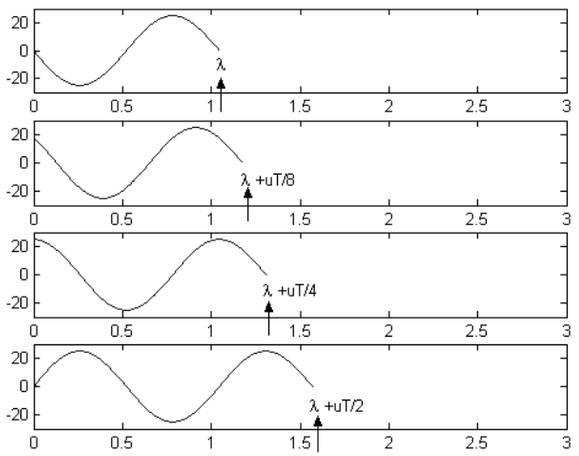


**Prob. 10.2**

(a)

$$\frac{\partial E}{\partial x} = -\sin(x+\omega t) - \sin(x-\omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -\cos(x+\omega t) - \cos(x-\omega t) = -E$$

$$\frac{\partial E}{\partial t} = -\omega\sin(x+\omega t) - \omega\sin(x-\omega t)$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2\cos(x+\omega t) - \omega^2\cos(x-\omega t) = -\omega^2 E$$

$$\frac{\partial^2 E}{\partial t^2} - u^2\frac{\partial^2 E}{\partial x^2} = -\omega^2 E + u^2 E = 0$$

if $\omega^2 = u^2$ and hence, eq. (10.1) is satisfied.

(b) $\underline{\underline{u = \omega}}$

**Prob. 10.3**

(a) $\omega = \underline{\underline{10^8 \text{ rad/s}}}$

(b) $\beta = \dfrac{\omega}{c} = \dfrac{10^8}{3\times 10^8} = \underline{\underline{0.333 \text{ rad/m}}}$

(c) $\lambda = \dfrac{2\pi}{\beta} = 6\pi = \underline{\underline{18.85 \text{ m}}}$

(d) Along $-\mathbf{a}_y$

At y=1, t=10ms,

(e) $H = 0.5\cos(10^8 t \times 10\times 10^{-9} + \dfrac{1}{3}\times 3) = 0.5\cos(1+1)$

$= \underline{\underline{-0.1665 \text{ A/m}}}$

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**Prob. 10.4**

(a) $\lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{60} = \underline{\underline{5\times10^6 \text{ m}}}$

(b) $\lambda = \dfrac{3\times10^8}{2\times10^6} = \underline{\underline{150 \text{ m}}}$

(c) $\lambda = \dfrac{3\times10^8}{120\times10^6} = \underline{\underline{2.5 \text{ m}}}$

(d) $\lambda = \dfrac{3\times10^8}{2.4\times10^9} = \underline{\underline{0.125 \text{ m}}}$

**Prob. 10.5** If

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = -\omega^2\mu\varepsilon + j\omega\mu\sigma$ and $\gamma = \alpha + j\beta$, then

$$|\gamma^2| = \sqrt{(\alpha^2 - \beta^2) + 4\alpha^2\beta^2} = \sqrt{(\alpha^2 + \beta^2)^2} = \alpha^2 + \beta^2$$

i.e.

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{(\sigma^2 + \omega^2\varepsilon^2)} \qquad (1)$$

$$\text{Re}(\gamma^2) = \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \qquad (2)$$

Subtracting and adding (1) and (2) lead respectively to

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

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(b) From eq. (10.25), $\boldsymbol{E}_s(z) = E_o e^{-\gamma z}\boldsymbol{a}_x$.

$$\nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = \frac{j}{\omega\mu}\nabla \times \boldsymbol{E}_s = \frac{j}{\omega\mu}(-\gamma E_o e^{-\gamma z}\boldsymbol{a}_y)$$

But $\boldsymbol{H}_s(z) = H_o e^{-\gamma z}\boldsymbol{a}_y$, hence $H_o = \frac{E_o}{\eta} = -\frac{j\gamma}{\omega\mu}E_o$

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From (b),

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}, \tan 2\theta_\eta = \left(\frac{\omega\varepsilon}{\sigma}\right)^{-1} = \frac{\sigma}{\omega\varepsilon}$$

**Prob. 10.6** (a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{8\times 10^{-2}}{2\pi \times 50\times 10^6 \times 3.6 \times \frac{10^{-9}}{36\pi}} = 8$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi\times 50\times 10^6}{3\times 10^8}\sqrt{\frac{2.1\times 3.6}{2}[\sqrt{65} - 1]} = 5.41$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} = 6.129$$

$$\gamma = \alpha + j\beta = \underline{\underline{5.41 + j6.129}}\ /\text{m}$$

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.129} = \underline{\underline{1.025}}\ \text{m}$

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(c) $u = \dfrac{\omega}{\beta} = \dfrac{2\pi \times 50 \times 10^6}{6.129} = \underline{\underline{5.125 \times 10^7}}$ m/s

(d) $|\eta| = \dfrac{\sqrt{\dfrac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}} = \dfrac{120\pi\sqrt{\dfrac{2.1}{3.6}}}{\sqrt[4]{65}} = 101.4$

$\tan 2\theta_\eta = \dfrac{\sigma}{\omega\varepsilon} = 8 \longrightarrow \theta_\eta = 41.44^o$

$\eta = \underline{\underline{101.41 \angle 41.44^o\ \Omega}}$

(e) $\boldsymbol{H}_s = \boldsymbol{a}_k \times \dfrac{\boldsymbol{E}_s}{\eta} = \boldsymbol{a}_x \times \dfrac{6}{\eta} e^{-\gamma z}\boldsymbol{a}_z = -\dfrac{6}{\eta} e^{-\gamma z}\boldsymbol{a}_y = \underline{\underline{-59.16 e^{-j41.44^o} e^{-\gamma z}\boldsymbol{a}_y}}$ mA/m

**Prob. 10.7**

(a) $\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-2}}{2\pi \times 12 \times 10^6 \times 10 \times \dfrac{10^{-9}}{36\pi}} = \underline{\underline{1.5}}$

(b) $\tan\theta = \dfrac{10^{-4}}{2\pi \times 12 \times 10^6 \times 4 \times \dfrac{10^{-9}}{36\pi}} = \underline{\underline{3.75 \times 10^{-2}}}$

(c) $\tan\theta = \dfrac{4}{2\pi \times 12 \times 10^6 \times 81 \times \dfrac{10^{-9}}{36\pi}} = \underline{\underline{74.07}}$

**Prob. 10.8**

(a)

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi \times 15 \times 10^9}{3 \times 10^8}\sqrt{\frac{1 \times 9.6}{2}\left[\sqrt{1 + 9 \times 10^{-8}} - 1\right]}$$

$$= 100\pi\sqrt{4.8\left(\frac{1}{2} \times 9 \times 10^{-8}\right)} = 0.146$$

$\delta = \dfrac{1}{\alpha} = \underline{\underline{6.85\ \text{m}}}$

(b) $A = \alpha\ell = 0.146 \times 5 \times 10^{-3} = \underline{\underline{0.73 \times 10^{-3}\ \text{Np}}}$

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**Prob. 10.9**

The phase difference is $\theta_\eta$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \frac{8\times10^{-3}}{2\pi\times20\times10^{6}\times4\times\dfrac{10^{-9}}{36\pi}} = 1.8$$

$$2\theta_\eta = 60.9^o \quad \longrightarrow \quad \theta_\eta = \underline{\underline{30.47^o}}$$

**Prob. 10.10**

For silver, the loss tangent is

$$\frac{\sigma}{\omega\varepsilon} = \frac{6.1\times10^{7}}{2\pi\times10^{8}\times\dfrac{10^{-9}}{36\pi}} = 6.1\times18\times10^{8} \gg 1$$

Hence, silver is a good conductor

For rubber,

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-15}}{2\pi\times10^{8}\times3.1\times\dfrac{10^{-9}}{36\pi}} = \frac{18}{3.1}\times10^{-14} \ll 1$$

Hence, rubber is a poor conductor or a good insulator.

**Prob. 10.11**

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi\times10^{5}\times80\times10^{-9}/36\pi} = 9{,}000 >> 1$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi\times10^{5}}{2}\times4\pi\times10^{-7}\times4} = 0.4\pi$$

(a) $u = \omega/\beta = \dfrac{2\pi\times10^{5}}{0.4\pi} = \underline{\underline{5\times10^{5}}}$ m/s

(b) $\lambda = 2\pi/\beta = \dfrac{2\pi}{0.4\pi} = \underline{\underline{5}}$ m

(c) $\delta = 1/\alpha = \dfrac{1}{0.4\pi} = \underline{\underline{0.796}}$ m

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(d) $\eta = |\eta| \angle \theta_\eta, \theta_\eta = 45^o$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} \cong \sqrt{\frac{\mu}{\varepsilon}\frac{\omega\varepsilon}{\sigma}} = \sqrt{\frac{4\pi\times10^{-7}\times2\pi\times10^5}{4}} = 0.4443$$

$$\eta = \underline{\underline{0.4443\angle 45^o \quad \Omega}}$$

**Prob. 10.12**

$$\frac{\sigma}{\omega\varepsilon} = \frac{1}{2\pi\times10^9\times4\times\frac{10^{-9}}{36\pi}} = 4.5$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$= 2\pi\times10^9\sqrt{\frac{4\pi}{2}\times10^{-7}\times4\times9\times\frac{10^{-9}}{36\pi}\left[\sqrt{1+4.5^2}-1\right]}$$

$$= 20\pi\sqrt{2[\sqrt{21.25}-1]} = \underline{\underline{168.8 \text{ Np/m}}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} = 20\pi\sqrt{2[\sqrt{21.25}+1]} = \underline{\underline{210.5 \text{ rad/m}}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 4.5 \quad \longrightarrow \quad \theta_\eta = 38.73^o$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi\sqrt{9/4}}{\sqrt[4]{1+4.5^2}} = 263.38$$

$$\eta = \underline{\underline{263.38\angle 38.73^o \quad \Omega}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi\times10^9}{210.5} = \underline{\underline{2.985\times10^7 \text{ m/s}}}$$

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**Prob. 10.13**

This is a lossy medium in which $\mu=\mu_o$.

Let $x = \left(\frac{\sigma}{\omega\varepsilon}\right)^2$

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j2\pi\times10^9\times4\pi}{100+j200} = 35.31\angle 26.57^o$$

$$E_o = 0.05\times35.31 = 1.765$$

$$\boldsymbol{a}_E = \boldsymbol{a}_H \times \boldsymbol{a}_k = -\boldsymbol{a}_z$$

Thus, we obtain

$$\boldsymbol{E} = \underline{\underline{-1.765\cos(2\pi\times10^9 t - 200x + 26.57^o)\boldsymbol{a}_z \text{ V/m}}}$$

$$\sqrt{\varepsilon_r(1/3)} = \frac{\alpha c}{\omega} = \frac{100\times3\times10^8}{2\pi\times10^9} = \frac{15}{\pi}$$

$$\varepsilon_r\frac{1}{3} = 4.776 \quad \longrightarrow \quad \varepsilon_r = 14.32$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \frac{4}{3} \quad \longrightarrow \quad \theta_\eta = 26.57^o$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+x}} = \frac{\frac{377}{\sqrt{14.32}}}{\sqrt{5/3}} = 77.175$$

$$E_o = |\eta| H_o = 77.175\times50\times10^{-3} = 3.858$$

$$\boldsymbol{a}_E = -(\boldsymbol{a}_k \times \boldsymbol{a}_H) = -(\boldsymbol{a}_x \times \boldsymbol{a}_y) = -\boldsymbol{a}_z$$

$$\boldsymbol{E} = \underline{\underline{-3.858e^{-100x}\cos(2\pi\times10^9 t - 200x + 26.57^o)\boldsymbol{a}_z \text{ V/m}}}$$

**Prob. 10.14** (a)

$$T = 1/f = 2\pi/\omega = \frac{2\pi}{\pi x10^8} = \underline{\underline{20 \text{ ns}}}$$

(b) Let $x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2}$$

But $\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}}\sqrt{x-1}$

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$$\sqrt{x-1} = \frac{\alpha c}{\omega\sqrt{\frac{\mu_r \varepsilon_r}{2}}} = \frac{0.1\times 3\times 10^8}{\pi\times 10^8\sqrt{2}} = 0.06752 \longrightarrow x = 1.0046$$

$$\beta = \left(\frac{x+1}{x-1}\right)^{1/2}\alpha = \left(\frac{2.0046}{0.0046}\right)^{1/2} 0.1 = 2.088$$

$$\lambda = 2\pi/\beta = \frac{2\pi}{2.088} = \underline{\underline{3}}\ \text{m}$$

(c) $$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x}} = \frac{377}{2\sqrt{1.0046}} = 188.1$$

$$x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} = 1.0046$$

$$\frac{\sigma}{\omega\varepsilon} = 0.096 = \tan 2\theta_\eta \longrightarrow \theta_\eta = 2.74^o$$

$$\eta = 188.1\angle 2.74^o \quad \Omega$$

$$E_o = \eta H_o = 12\times 188.1 = 2257.2$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_E \times \boldsymbol{a}_x = \boldsymbol{a}_y \longrightarrow \boldsymbol{a}_E = \boldsymbol{a}_z$$

$$\boldsymbol{E} = \underline{\underline{2.257e^{-0.1y}\sin(\pi\times 10^8 t - 2.088y + 2.74^o)\boldsymbol{a}_z}}\ \text{kV/m}$$

(d) The phase difference is $\underline{\underline{2.74^o}}$.

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**Prob. 10.15**

(a) $\beta = 6.5 = \omega\sqrt{\mu_o \varepsilon_o} = \dfrac{\omega}{c}$

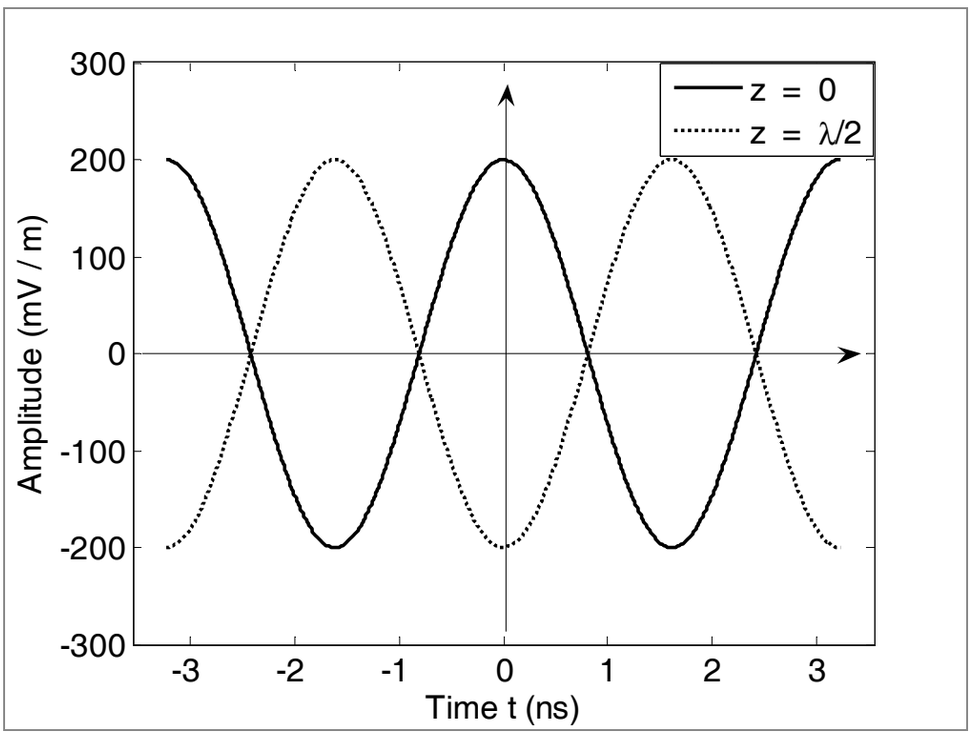
$\omega = \beta c = 6.5 \times 3 \times 10^8 = \underline{\underline{1.95 \times 10^9 \text{ rad/s}}}$

$\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{6.5} = \underline{\underline{0.9666 \text{ m}}}$

(b) For z=0, $E_z = 0.2\cos\omega t$

For z=$\lambda$/2, $E_z = 0.2\cos(\omega t - \dfrac{2\pi}{\lambda}\dfrac{\lambda}{2}) = -0.2\cos\omega t$

The two waves are sketched below.




(c) $\boldsymbol{H} = H_o \cos(\omega t - 6.5z)\boldsymbol{a}_H$

$H_o = \dfrac{E_o}{\eta_o} = \dfrac{0.2}{377} = 5.305 \times 10^{-4}$

$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \quad \longrightarrow \quad \boldsymbol{a}_x \times \boldsymbol{a}_H = \boldsymbol{a}_z \quad \longrightarrow \quad \boldsymbol{a}_H = \boldsymbol{a}_y$

$\boldsymbol{H} = \underline{\underline{0.5305\cos(\omega t - 6.5z)\boldsymbol{a}_y \text{ mA/m}}}$

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**Prob. 10.16**

$$u = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{3\times10^8}{\sqrt{3\times4}} = \underline{\underline{8.66\times10^7 \text{ m/s}}}$$

$$\lambda = \frac{u}{f} = \frac{8.66\times10^7}{60\times10^6} = \underline{\underline{1.443 \text{ m}}}$$

$$\eta = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = 377\sqrt{\frac{4}{3}} = \underline{\underline{435.32\ \Omega}}$$

**Prob. 10.17** (a) Along <u>-x direction.</u>

(b) $\beta = 6, \quad \omega = 2\times10^8$,

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \beta c / \omega = \frac{6\times3\times10^8}{2\times10^8} = 9 \quad \longrightarrow \quad \varepsilon_r = 81$$

$$\varepsilon = \varepsilon_o\varepsilon_r = \frac{10^{-9}}{36\pi}\times81 = \underline{\underline{7.162\times10^{-10}}} \text{ F/m}$$

(c) $\eta = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{\mu_r/\varepsilon_r} = \frac{120\pi}{9} = 41.89\ \Omega$

$$E_o = H_o\eta = 25\times10^{-3}\times41.88 = 1.047$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_E \times \boldsymbol{a}_y = -\boldsymbol{a}_x \longrightarrow \boldsymbol{a}_E = \boldsymbol{a}_z$$

$$\boldsymbol{E} = \underline{\underline{1.047\sin(2\times10^8 t + 6x)\boldsymbol{a}_z}} \text{ V/m}$$

**Prob. 10.18** (a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-6}}{2\pi\times10^7\times5\times\dfrac{10^{-9}}{36\pi}} = 3.6\times10^{-4} << 1$

Thus, the material is <u>lossless</u> at this frequency.

(b) $\beta = \omega\sqrt{\mu\varepsilon} = \dfrac{2\pi\times10^7}{3\times10^8}\sqrt{5\times750} = \underline{\underline{12.83}}$ rad/m

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$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12.83} = \underline{\underline{0.49}} \text{ m}$$

(c) Phase difference = $\beta l = \underline{\underline{25.66 \text{ rad}}}$

(d) $\eta = \sqrt{\mu / \varepsilon} = 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi \sqrt{\frac{750}{5}} = \underline{\underline{4.62 \text{ k}\Omega}}$

**Prob. 10.19**

(a) $\boldsymbol{E} = \text{Re}[\boldsymbol{E}_s e^{j\omega t}] = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y) e^{-0.2z} \cos(\omega t - 3.4z)$

At $z = 4\text{m}$, $t = T/8$, $\omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$$\boldsymbol{E} = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y) e^{-0.8} \cos(\pi / 4 - 13.6)$$

$$|E| = 13e^{-0.8} |\cos(\pi / 4 - 13.6)| = \underline{\underline{5.662}} \text{ V/m}$$

(b) loss $= \alpha \Delta z = 0.2(3) = 0.6$ Np. Since 1 Np = 8.686 dB,

loss = 0.6 x 8.686 = $\underline{\underline{5.212 \text{ dB}}}$

(c) Let $x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2 / 3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega\sqrt{\mu\varepsilon / 2}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r / 2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2 \times 3 \times 10^8}{10^8\sqrt{0.00694}} = 7.2 \quad \longrightarrow \quad \varepsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o}} \cdot \frac{1}{\sqrt{\varepsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68 \times 1.00694}} = 36.896$$

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$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2 - 1} = 0.118 \quad \longrightarrow \quad \theta_\eta = 3.365^o$$

$$\eta = \underline{\underline{36.896\angle 3.365^o\ \Omega}}$$

**Prob. 10.20**

This is a lossless material.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 377\sqrt{\frac{\mu_r}{\varepsilon_r}} = 105 \qquad (1)$$

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = 7.6\times 10^7 \qquad (2)$$

From (1),

$$\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{105}{377} = 0.2785 \qquad (1)a$$

From (2),

$$\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{7.6\times 10^7}{3\times 10^8} = 0.2533 \qquad (2)a$$

Multiplying (1)a by (2)a,

$$\frac{1}{\varepsilon_r} = 0.2785\times 0.2533 = 0.07054 \quad \longrightarrow \quad \varepsilon_r = \underline{\underline{14.175}}$$

Dividing (1)a by (2)a,

$$\mu_r = \frac{0.2785}{0.2533} = \underline{\underline{1.0995}}$$

**Prob. 10.21**

(a)
$$\nabla\times\boldsymbol{E} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x(z,t) & E_y(z,t) & 0 \end{vmatrix} = -\frac{\partial E_y}{\partial z}\boldsymbol{a}_x + \frac{\partial E_x}{\partial z}\boldsymbol{a}_y$$

$$= -6\beta\cos(\omega t - \beta z)\boldsymbol{a}_x + 8\beta\sin(\omega t - \beta z)\boldsymbol{a}_y$$

But $\nabla\times\boldsymbol{E} = -\mu\dfrac{\partial \boldsymbol{H}}{\partial t} \quad \longrightarrow \quad \boldsymbol{H} = -\dfrac{1}{\mu}\int \nabla\times\boldsymbol{E}\,dt$

$$\boldsymbol{H} = \underline{\underline{\frac{6\beta}{\mu\omega}\sin(\omega t - \beta z)\boldsymbol{a}_x + \frac{8\beta}{\mu\omega}\cos(\omega t - \beta z)\boldsymbol{a}_y}}$$

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(b) $\beta = \omega\sqrt{\mu\varepsilon} = \dfrac{2\pi f}{c}\sqrt{4.5} = \dfrac{2\pi \times 40\times 10^6}{3\times 10^8}\sqrt{4.5} = \underline{\underline{1.777 \text{ rad/m}}}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.777} = \underline{\underline{3.536 \text{ m}}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{4.5}} = \underline{\underline{177.72\ \Omega}}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{4.5}} = \frac{3\times 10^8}{\sqrt{4.5}} = \underline{\underline{1.4142\times 10^8 \text{ m/s}}}$$

**Prob. 10.22**

$$0.4E_o = E_o e^{-\alpha z} \quad \longrightarrow \quad \frac{1}{0.4} = e^{2\alpha}$$

Or $\alpha = \dfrac{1}{2}\ln\dfrac{1}{0.4} = 0.4581 \quad \longrightarrow \quad \delta = 1/\alpha = \underline{\underline{2.183}}$ m

$$\lambda = 2\pi/\beta = 2\pi/1.6$$

$$u = f\lambda = 10^7 \times \frac{2\pi}{1.6} = \underline{\underline{3.927\times 10^7}} \text{ m/s}$$

**Prob. 10.23**

(a)

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{10^8\pi}{3\times 10^8} = \frac{\pi}{3} = \underline{\underline{1.0472 \text{ rad/m}}}$$

(b)

$$E = 0 \quad \longrightarrow \quad \sin(10^8\pi t_o - \beta x_o) = 0 = \sin(n\pi), n = 1,2,3,...$$

$$10^8\pi t_o - \beta x_o = \pi$$

$$10^8\pi \times 5\times 10^{-3} - \frac{\pi}{3}x_o = \pi \quad \longrightarrow \quad x_o \simeq \underline{\underline{5\times 10^5 \text{ m}}}$$

(c)

$$\boldsymbol{H} = H_o \sin(10^8\pi t - \beta x)\boldsymbol{a}_H$$

$$H_o = \frac{E_o}{\eta} = \frac{50\times 10^{-3}}{120\pi} = 132.63\ \mu\text{A/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_x \times \boldsymbol{a}_z = -\boldsymbol{a}_y$$

$$\underline{\underline{\boldsymbol{H} = -132.63\sin(10^8\pi t - 1.0472x)\boldsymbol{a}_y\ \mu\text{A/m}}}$$

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**Prob. 10.24**

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \longrightarrow \quad \sigma = \frac{2\alpha^2}{\omega\mu} = \frac{2\times 12^2}{2\pi\times 10^6 \times 4\pi\times 10^{-7}} = \underline{\underline{36.48}}$$

$$\eta = |\eta| \angle\theta_\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^o$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{2\pi\times 10^6 \times 4\pi\times 10^{-7}}{36.48}} = 0.4652$$

$$E_o = |\eta| H_o = 0.4652\times 20\times 10^{-3} = 9.305\times 10^{-3}$$

$$\boldsymbol{a}_E = \boldsymbol{a}_H \times \boldsymbol{a}_k = \boldsymbol{a}_y \times (-\boldsymbol{a}_z) = -\boldsymbol{a}_x$$

$$\boldsymbol{E} = E_o e^{-\alpha z} \sin(\omega t + \beta z)\boldsymbol{a}_E$$

$$= -9.305 e^{-12z} \sin(2\pi\times 10^6 t + 12z + 45^o)\boldsymbol{a}_x \text{ mV/m}$$

**Prob. 10.25** For a good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$, say $\frac{\sigma}{\omega\varepsilon} > 100$

(a) $$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{2\pi\times 8\times 10^6 \times 15\times \frac{10^{-9}}{36\pi}} = 1.5 \quad \longrightarrow \quad \text{lossy}$$

No, not conducting.

(b) $$\frac{\sigma}{\omega\varepsilon} = \frac{0.025}{2\pi\times 8\times 10^6 \times 16\times \frac{10^{-9}}{36\pi}} = 3.515 \quad \longrightarrow \quad \text{lossy}$$

No, not conducting.

(c) $$\frac{\sigma}{\omega\varepsilon} = \frac{25}{2\pi\times 8\times 10^6 \times 81\times \frac{10^{-9}}{36\pi}} = 694.4 \quad \longrightarrow \quad \text{conducting}$$

Yes, conducting.

**Prob. 10.26**

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{2\pi f}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1.0049} - 1\right]} = \frac{2\pi\times 6\times 10^6}{3\times 10^8}\sqrt{\frac{4}{2}\times 2.447\times 10^{-3}}$$

$$\alpha = 8.791\times 10^{-3}$$

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$$\delta = 1/\alpha = \underline{\underline{113.75 \text{ m}}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} = \frac{4\pi}{100}\sqrt{\frac{4}{2}\left[\sqrt{1.0049}+1\right]} = 0.2515$$

$$u = \omega/\beta = \frac{2\pi\times 6\times 10^6}{0.2515} = \underline{\underline{1.5\times 10^8}} \text{ m/s}$$

**Prob. 10.27** (a)

$$R_{dc} = \frac{l}{\sigma S} = \frac{l}{\sigma\pi a^2} = \frac{600}{5.8\times 10^7\times\pi\times(1.2)^2\times 10^{-6}} = \underline{\underline{2.287\ \Omega}}$$

(b) $R_{ac} = \dfrac{l}{\sigma 2\pi a\delta}$. At 100 MHz, $\delta = 6.6\times 10^{-3}$ mm =$6.6\times 10^{-6}$ m mm for copper (see Table 10.2).

$$R_{ac} = \frac{600}{5.8\times 10^7\times 2\pi\times(1.2\times 10^{-3})\times 6.6\times 10^{-6}} = \underline{\underline{207.61\ \Omega}}$$

(c) $$\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = 1 \quad\longrightarrow\quad \delta = a/2 = \frac{66.1\times 10^{-3}}{\sqrt{f}}$$

$$\sqrt{f} = \frac{66.1\times 2\times 10^{-3}}{a} = \frac{66.1\times 2}{1.2} \quad\longrightarrow\quad f = \underline{\underline{12.137}} \text{ kHz}$$

**Prob. 10.28**

(a) $$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{3.5\times 10^7}{2\pi\times 150\times 10^6\times\frac{10^{-9}}{36\pi}} = \frac{3.5\times 18\times 10^9}{15} >> 1$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma} = \sqrt{150\pi\times 10^6\times 4\pi\times 10^{-7}\times 3.5\times 10^7} = 143{,}965.86$$

$$\gamma = \alpha + j\beta = \underline{\underline{1.44(1+j)\times 10^5 \text{ /m}}}$$

(b) $$\delta = 1/\alpha = \underline{\underline{6.946\times 10^{-6} \text{ m}}}$$

(c) $$u = \frac{\omega}{\beta} = \frac{150\times 2\pi\times 10^6}{1.44\times 10^5} = \underline{\underline{6547 \text{ m/s}}}$$

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**Prob. 10.29**

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi \times 2 \times 10^9 \times 24 \times \frac{10^{-9}}{36\pi}} = 1.5$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} = 2\pi \times 2 \times 10^9 \sqrt{\frac{4\pi \times 10^{-7} \times 24 \times \frac{10^{-9}}{36\pi}}{2}\left[\sqrt{1+(1.5)^2}-1\right]}$$

$$= 130.01 \text{ Np/m}$$

$10^{-5} E_o = E_o e^{-\alpha d}$

Taking the log of both sides gives

$$-5\ln 10 = -\alpha d \quad \longrightarrow \quad d = \frac{5\ln 10}{\alpha} = \frac{5\ln 10}{130.01} = \underline{\underline{0.0886 \text{ m}}}$$

**Prob. 10.30**

$\alpha = \beta = 1/\delta$

$\lambda = 2\pi / \beta = 2\pi\delta = 6.283\delta \quad \longrightarrow \quad \delta = 0.1591\lambda$

showing that $\delta$ is shorter than $\lambda$.

**Prob. 10.31**

$$t = 5\delta = \frac{5}{\sqrt{\pi f \mu \sigma}} = \frac{5}{\sqrt{\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7} \times 6.1 \times 10^7}} = \underline{\underline{2.94 \times 10^{-6} \text{ m}}}$$

**Prob. 10.32**

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad \longrightarrow \quad f = \frac{1}{\delta^2 \pi \mu \sigma}$$

$$f = \frac{1}{4 \times 10^{-6} \times \pi \times 4\pi \times 10^{-7} \times 6.1 \times 10^7} = \underline{\underline{1.038 \text{ kHz}}}$$

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**Prob. 10.33**

(a) Linearly polarized along $\mathbf{a}_z$

(b) $\omega = 2\pi f = 2\pi \times 10^7 \quad \longrightarrow \quad f = 10^7 = \underline{\underline{10 \text{ MHz}}}$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

(c)

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{3\times 3\times 10^8}{2\pi\times 10^7} = 14.32 \quad \longrightarrow \quad \varepsilon_r = \underline{\underline{205.18}}$$

Let $\boldsymbol{H} = H_o \sin(\omega t - 3y)\boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta}, \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{\sqrt{\varepsilon_r}} = \frac{120\pi}{14.32} = 26.33$$

(d) $$H_o = \frac{12}{26.33} = 0.456$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_y \times \boldsymbol{a}_z = \boldsymbol{a}_x$$

$$\boldsymbol{H} = \underline{\underline{0.456\sin(2\pi\times 10^7 t - 3y)\boldsymbol{a}_x \text{ A/m}}}$$

**Prob. 10.34**

$$\boldsymbol{E} = (2\boldsymbol{a}_y - 5\boldsymbol{a}_z)\sin(\omega t - \beta x)$$

The ratio $E_y / E_z$ remains the same as t changes. Hence the wave is <u>linearly</u> polarized

**Prob. 10.35**

(a)

$$E_x = E_o \cos(\omega t + \beta y), \qquad E_y = E_o \sin(\omega t + \beta y)$$

$$E_x(0,t) = E_o \cos\omega t \quad \longrightarrow \quad \cos\omega t = \frac{E_x(0,t)}{E_o}$$

$$E_y(0,t) = E_o \sin\omega t \quad \longrightarrow \quad \sin\omega t = \frac{E_y(0,t)}{E_o}$$

$$\cos^2\omega t + \sin^2\omega t = 1 \quad \longrightarrow \quad \left(\frac{E_x}{E_o}\right)^2 + \left(\frac{E_y}{E_o}\right)^2 = 1$$

Hence, we have <u>circular polarization</u>.

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(b)

$E_x = E_o \cos(\omega t - \beta y), \quad E_y = -3E_o \sin(\omega t - \beta y)$

In the y=0 plane,

$$E_x(0,t) = E_o \cos \omega t \quad \longrightarrow \quad \cos \omega t = \frac{E_x(0,t)}{E_o}$$

$$E_y(0,t) = E_o \sin \omega t \quad \longrightarrow \quad \sin \omega t = \frac{-E_y(0,t)}{3E_o}$$

$$\cos^2 \omega t + \sin^2 \omega t = 1 \quad \longrightarrow \quad \left(\frac{E_x}{E_o}\right)^2 + \frac{1}{9}\left(\frac{E_y}{E_o}\right)^2 = 1$$

Hence, we have elliptical polarization.

**Prob. 10.36**

(a) We can write

$\boldsymbol{E} = \text{Re}(E_s e^{j\omega t}) = (40\boldsymbol{a}_x + 60\boldsymbol{a}_y)\cos(\omega t - 10z)$

Since $E_x / E_y$ does not change with time, the wave is linearly polarized.

(b) This is elliptically polarized.

**Prob. 10.37**

(a)

When $\phi = 0$,

$\boldsymbol{E}(y,t) = (E_{o1}\boldsymbol{a}_x + E_{o2}\boldsymbol{a}_z)\cos(\omega t - \beta y)$

The two components are in phase and the wave is linearly polarized.

(b)

When $\phi = \pi / 2$,

$E_z = E_{o2}\cos(\omega t - \beta y + \pi / 2) = -E_{o2}\sin(\omega t - \beta y)$

We can combine $E_x$ and $E_z$ to show that the wave is elliptically polarized.

(c)

When $\phi = \pi$,

$$\boldsymbol{E}(y,t) = E_{o1}\cos(\omega t - \beta y)\boldsymbol{a}_x + E_{o2}\cos(\omega t - \beta y + \pi)\boldsymbol{a}_z$$
$$= (E_{o1}\boldsymbol{a}_x - E_{o2}\boldsymbol{a}_y)\cos(\omega t - \beta y)$$

Thus, the wave is linearly polarized.

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**Prob. 10.38**

We can write $\mathbf{E}_s$ as

$$\boldsymbol{E}_s = \boldsymbol{E}_1(z) + \boldsymbol{E}_2(z)$$

where

$$\boldsymbol{E}_1(z) = \frac{1}{2} E_o(\boldsymbol{a}_x - j\boldsymbol{a}_y) e^{-j\beta z}$$

$$\boldsymbol{E}_2(z) = \frac{1}{2} E_o(\boldsymbol{a}_x + j\boldsymbol{a}_y) e^{-j\beta z}$$

We recognize that $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are circularly polarized waves. The problem is therefore proved.

**Prob. 10.39**

(a) The wave is elliptically polarized.

(b)

Let $\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2$,

where $\boldsymbol{E}_1 = 40\cos(\omega t - \beta z)\boldsymbol{a}_x$, $\boldsymbol{E}_2 = 60\sin(\omega t - \beta z)\boldsymbol{a}_y$

$$\boldsymbol{H}_1 = H_{o1}\cos(\omega t - \beta z)\boldsymbol{a}_{H1}$$

$$H_{o1} = \frac{40}{\eta_o} = \frac{40}{120\pi} = 0.106$$

$$\boldsymbol{a}_{H1} = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_z \times \boldsymbol{a}_x = \boldsymbol{a}_y$$

$$\boldsymbol{H}_1 = 0.106\cos(\omega t - \beta z)\boldsymbol{a}_y$$

$$\boldsymbol{H}_2 = H_{o2}\sin(\omega t - \beta z)\boldsymbol{a}_{H2}$$

$$H_{o1} = \frac{60}{\eta_o} = \frac{60}{120\pi} = 0.1592$$

$$\boldsymbol{a}_{H2} = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_z \times \boldsymbol{a}_y = -\boldsymbol{a}_x$$

$$\boldsymbol{H}_2 = -0.1592\sin(\omega t - \beta z)\boldsymbol{a}_x$$

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 = \underline{\underline{-159.2\sin(\omega t - \beta z)\boldsymbol{a}_x + 106\cos(\omega t - \beta z)\boldsymbol{a}_y \text{ mA/m}}}$$

**Prob. 10.40**

Let $\boldsymbol{E}_s = \boldsymbol{E}_r + j\boldsymbol{E}_i$ and $\boldsymbol{H}_s = \boldsymbol{H}_r + j\boldsymbol{H}_i$

$$\boldsymbol{E} = \text{Re}(\boldsymbol{E}_s e^{j\omega t}) = \boldsymbol{E}_r \cos\omega t - \boldsymbol{E}_i \sin\omega t$$

Similarly,

$$\boldsymbol{H} = \boldsymbol{H}_r \cos\omega t - \boldsymbol{H}_i \sin\omega t$$

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$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H} = \boldsymbol{E}_r \times \boldsymbol{H}_r \cos^2 \omega t + \boldsymbol{E}_i \times \boldsymbol{H}_i \sin^2 \omega t - \frac{1}{2}(\boldsymbol{E}_r \times \boldsymbol{H}_i + \boldsymbol{E}_i \times \boldsymbol{H}_r) \sin\ 2\omega t$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{T}\int_0^T \mathcal{P} dt = \frac{1}{T}\int_0^T \cos^2 \omega dt (\boldsymbol{E}_r \times \boldsymbol{H}_r) + \frac{1}{T}\int_0^T \sin^2 \omega dt (\boldsymbol{E}_i \times \boldsymbol{H}_i) - \frac{1}{2T}\int_0^T \sin 2\omega dt (\boldsymbol{E}_i \times \boldsymbol{H}_i + \boldsymbol{E}_i \times \boldsymbol{H}_r)$$

$$= \frac{1}{2}(\boldsymbol{E}_r \times \boldsymbol{H}_r + \boldsymbol{E}_i \times \boldsymbol{H}_i) = \frac{1}{2}\text{Re}[(\boldsymbol{E}_r + j\boldsymbol{E}_i) \times (\boldsymbol{H}_r - j\boldsymbol{H}_i)]$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\ \text{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s^*)$$

as required.

**Prob. 10.41**

(a)

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o \varepsilon_o}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{8 \times 3 \times 10^8}{10^9} = 2.4$$

$$\varepsilon_r = \underline{\underline{5.76}}$$

(b) $$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\frac{1}{\sqrt{\varepsilon_r}} = \frac{377}{2.4} = \underline{\underline{157.1\ \Omega}}$$

(c) $$u = \frac{\omega}{\beta} = \frac{10^9}{8} = \underline{\underline{1.25 \times 10^8\ \text{m/s}}}$$

(d)

Let $$\boldsymbol{H} = H_o \cos(10^9 t + 8x)\boldsymbol{a}_H$$

$$H_o = \frac{E_o}{\eta} = \frac{150}{157.1} = 0.955$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = -\boldsymbol{a}_x \times \boldsymbol{a}_z = \boldsymbol{a}_y$$

$$\boldsymbol{H} = \underline{\underline{0.955\cos(10^9 t + 8x)\boldsymbol{a}_y\ \text{A/m}}}$$

(e)

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H} = \text{-}150(0.955)\cos^2(10^9 t + 8x)\boldsymbol{a}_x$$

$$= \underline{\underline{\text{-}143.25\cos^2(10^9 t + 8x)\boldsymbol{a}_x\ \text{W/m}^2}}$$

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**Prob. 10.42**

$$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} = \begin{vmatrix} E_x & 0 & E_z \\ 0 & H_y & 0 \end{vmatrix} = -E_z H_y \boldsymbol{a}_x + E_x H_y \boldsymbol{a}_z$$

$$= -H_o E_{o2} \sin \alpha x \cos \alpha x \sin(\omega t - \beta x) \cos(\omega t - \beta z) \boldsymbol{a}_x + H_o E_{o1} \cos^2 \alpha x \cos^2(\omega t - \beta z) \boldsymbol{a}_z$$

$$= -\frac{1}{4} H_o E_{o2} \sin 2\alpha x \sin 2(\omega t - \beta x) \boldsymbol{a}_x + H_o E_{o1} \cos^2 \alpha x \cos^2(\omega t - \beta z) \boldsymbol{a}_z$$

$$\boldsymbol{P}_{ave} = \frac{1}{T} \int_0^T \boldsymbol{P} dt = 0 + \frac{1}{2} H_o E_{o1} \cos^2 \alpha x \boldsymbol{a}_z$$

$$= \underline{\underline{\frac{1}{2} E_{o1} H_o \cos^2 \alpha x \boldsymbol{a}_z}}$$

**Prob. 10.43**

(a)

Let $\boldsymbol{H}_s = \frac{H_o}{r} \sin\theta e^{-j3r} \boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = \frac{1}{12\pi}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_r \times \boldsymbol{a}_\theta = \boldsymbol{a}_\phi$$

$$\boldsymbol{H}_s = \underline{\underline{\frac{1}{12\pi r} \sin\theta e^{-j3r} \boldsymbol{a}_\phi \text{ A/m}}}$$

(b)

$$\mathcal{P}_{ave} = \frac{1}{2} \text{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s) = \frac{10}{2 \times 12\pi r^2} \sin^2\theta \boldsymbol{a}_r$$

$$P_{ave} = \int_S \mathcal{P}_{ave} \cdot d\boldsymbol{S}, \quad d\boldsymbol{S} = r^2 \sin\theta d\theta d\phi \boldsymbol{a}_r$$

$$P_{ave} = \frac{10}{24\pi} \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi/6} r^2 \sin^3\theta d\theta \Big|_{r=2} = \frac{5}{8} - \frac{5\sqrt{3}}{32} = 0.007145$$

$$= \underline{\underline{7.145 \text{ mW}}}$$

**Prob. 10.44**

(a) $P_{ave} = \frac{1}{2} \text{Re}(E_s H_s^*) = \frac{1}{2} \text{Re}(\frac{|E_s|}{|\eta|}) = \frac{8^2}{2|\eta|} e^{-0.2z}$

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$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

Let $\mathrm{x} = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = 0.1/0.3 = 1/3$$

$$\frac{x-1}{x+1} = \frac{1}{9} \quad \longrightarrow \quad x = 5/4$$

$$\frac{5}{4} = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \quad \longrightarrow \quad \frac{\sigma}{\omega\varepsilon} = 3/4$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi/\sqrt{81}}{\sqrt{\frac{5}{4}}} = 37.4657$$

$$P_{ave} = \frac{64}{2(37.4657)}e^{-0.2z} = \underline{\underline{0.8541e^{-0.2z}\ \mathrm{W/m^2}}}$$

(b) $20dB = 10\log\frac{P_1}{P_2} \quad \longrightarrow \quad \frac{P_1}{P_2} = 100$

$$\frac{P_2}{P_1} = e^{-0.2z} = \frac{1}{100} \quad \longrightarrow \quad e^{0.2z} = 100$$

$$z = 5\log 100 = \underline{\underline{23\ \mathrm{m}}}$$

**Prob. 10.45**

(a) $u = \omega/\beta \quad \longrightarrow \quad \omega = u\beta = \frac{\beta c}{\sqrt{4.5}} = \frac{2\times 3\times 10^8}{\sqrt{4.5}} = \underline{\underline{2.828\times 10^8}}$ rad/s

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7\Omega$$

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$$\boldsymbol{H} = \boldsymbol{a}_k \times \frac{\boldsymbol{E}}{\eta} = \frac{\boldsymbol{a}_z}{\eta} \times \frac{40}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\rho = \underline{\underline{\frac{0.225}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\phi}} \text{ A/m}$$

(b) $$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} = \underline{\underline{\frac{9}{\rho^2}\sin^2(\omega t - 2z)\boldsymbol{a}_z}} \text{ W/m}^2$$

(c) $$\mathcal{P}_{\text{ave}} = \frac{4.5}{\rho^2}\boldsymbol{a}_z, \; \mathrm{d}\mathbf{S} = \rho \mathrm{d}\phi \mathrm{d}\rho \boldsymbol{a}_z$$

$$\mathrm{P}_{\text{ave}} = \int \mathbf{P}_{\text{ave}} \bullet dS = 4.5 \int_{2\text{mm}}^{3\text{mm}} \frac{\mathrm{d}\rho}{\rho} \int_0^{2\pi} \mathrm{d}\phi = 4.5\ln(3/2)(2\pi) = \underline{\underline{11.46 \text{ W}}}$$

**Prob. 10.46**

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H} = \frac{E_o^2}{\eta r^2}\sin^2\theta \sin^2\omega(t - r/c)\boldsymbol{a}_r$$

$$\boldsymbol{P}_{ave} = \frac{1}{T}\int_0^T \mathcal{P}\, dt = \underline{\underline{\frac{E_o^2}{2\eta r^2}\sin^2\theta \boldsymbol{a}_r}}$$

**Prob. 10.47**

$$\beta = \frac{\omega}{c} \quad \longrightarrow \quad \omega = \beta c = 40(3\times 10^8) = \underline{\underline{12\times 10^9 \text{ rad/s}}}$$

$$\boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt$$

$$\nabla \times \boldsymbol{H} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 10\sin(\omega t - 40x) & -20\sin(\omega t - 40x) \end{vmatrix}$$

$$= -800\cos(\omega t - 40x)\boldsymbol{a}_y - 400\cos(\omega t - 40x)\boldsymbol{a}_z$$

$$\boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla \times \boldsymbol{H} dt = -\frac{800}{\omega\varepsilon}\sin(\omega t - 40x)\boldsymbol{a}_y - \frac{400}{\omega\varepsilon}\sin(\omega t - 40x)\boldsymbol{a}_z$$

$$= -\frac{800}{12\times 10^9 \times \dfrac{10^{-9}}{36\pi}}\sin(\omega t - 40x)\boldsymbol{a}_y - \frac{400}{12\times 10^9 \times \dfrac{10^{-9}}{36\pi}}\sin(\omega t - 40x)\boldsymbol{a}_z$$

$$= \underline{\underline{-7.539\sin(\omega t - 40x)\boldsymbol{a}_y - 3.77\sin(\omega t - 40x)\boldsymbol{a}_z \text{ kV/m}}}$$

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$$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} = \begin{vmatrix} 0 & E_y & E_z \\ 0 & H_y & H_z \end{vmatrix} = (E_y H_z - E_z H_y)\boldsymbol{a}_x$$

$$= \left[ 20(7.537)\sin^2(\omega t - 40x) + 37.7\sin^2(\omega t - 40x) \right] \boldsymbol{a}_x 10^3$$

$$\boldsymbol{P}_{\text{ave}} = \frac{1}{2}\left[20(7.537) + 37.7\right]\boldsymbol{a}_x 10^3 = \underline{\underline{94.23\,\boldsymbol{a}_x \text{ kW/m}^2}}$$

**Prob. 10.48**

$$P = \frac{E_o^2}{2\eta_o} \quad \longrightarrow \quad E_o^2 = 2\eta_o P = 2(120\pi)10 \times 10^{-3} = 7.539$$

$$E_o = \underline{\underline{2.746 \text{ V/m}}}$$

**Prob. 10.49**

Let $T = \omega t - \beta z$.

$$-\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{E} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \cos T & \sin T & 0 \end{vmatrix}$$

$$-\mu \frac{\partial \boldsymbol{H}}{\partial t} = \beta \cos T \boldsymbol{a}_x + \beta \sin T \boldsymbol{a}_y$$

$$\boldsymbol{H} = -\frac{\beta}{\mu}\int \left[ \cos T \boldsymbol{a}_x + \sin T \boldsymbol{a}_y \right] dt = -\frac{\beta}{\mu\omega}\sin T \boldsymbol{a}_x + \frac{\beta}{\mu\omega}\cos T \boldsymbol{a}_y$$

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H} = \begin{vmatrix} cosT & sinT & 0 \\ -\dfrac{\beta}{\mu\omega}\sin T & \dfrac{\beta}{\mu\omega}\cos T & 0 \end{vmatrix} = \frac{\beta}{\mu\omega}(\cos^2 T + \sin^2 T)\boldsymbol{a}_z$$

$$= \frac{\beta}{\mu\omega}\boldsymbol{a}_z = \sqrt{\frac{\varepsilon}{\mu}}\boldsymbol{a}_z$$

which is constant everywhere.

**Prob. 10.50**

$$\mathcal{P} = \frac{E_o^2}{2\eta_o}$$

$$P = \mathcal{P}S = \frac{E_o^2 S}{2\eta_o} = \frac{(2.4 \times 10^3)^2 \times 450 \times 10^{-4}}{2 \times 377} = \underline{\underline{343.8 \text{ W}}}$$

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**Prob. 10.51**

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H} = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \sin^2(\omega t - \beta z)\boldsymbol{a}_z$$

(a) $$\mathcal{P}_{ave} = \frac{1}{T}\int_0^T \mathcal{P}\, dt = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \frac{1}{T}\int_0^T \sin^2(\omega t - \beta z) dt \boldsymbol{a}_z = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \frac{1}{2}\boldsymbol{a}_z$$

$$= \underline{\underline{\frac{V_o I_o}{4\pi\rho^2 \ln(b/a)}\boldsymbol{a}_z}}$$

(b)

$$P_{ave} = \int_S \mathcal{P}_{ave} \cdot d\boldsymbol{S}, \qquad d\boldsymbol{S} = \rho\, d\rho d\phi \boldsymbol{a}_z$$

$$= \frac{V_o I_o}{4\pi \ln(b/a)} \int_{\phi=0}^{2\pi}\int_{\rho=a}^{b} \frac{1}{\rho^2}\rho d\rho d\phi = \frac{V_o I_o}{4\pi \ln(b/a)}(2\pi)\ln(b/a)$$

$$= \underline{\underline{\frac{1}{2}V_o I_o}}$$

**Prob. 10.52**

(a) $$P_{i,ave} = \frac{E_{io}^{\ 2}}{2\eta_1}, \qquad P_{r,ave} = \frac{E_{ro}^{\ 2}}{2\eta_1}, \qquad P_{t,ave} = \frac{E_{to}^{\ 2}}{2\eta_2}$$

$$R = \frac{P_{r,ave}}{P_{i,ave}} = \frac{E_{ro}^{\ 2}}{E_{io}^{\ 2}} = \Gamma^2 = \underline{\underline{\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)^2}}$$

$$R = \left(\frac{\sqrt{\frac{\mu_o}{\varepsilon_2}} - \sqrt{\frac{\mu_o}{\varepsilon_1}}}{\sqrt{\frac{\mu_o}{\varepsilon_2}} + \sqrt{\frac{\mu_o}{\varepsilon_1}}}\right)^2 = \left(\frac{\sqrt{\mu_o\varepsilon_1} - \sqrt{\mu_o\varepsilon_2}}{\sqrt{\mu_o\varepsilon_1} + \sqrt{\mu_o\varepsilon_2}}\right)^2$$

Since $n_1 = c\sqrt{\mu_1\varepsilon_1} = c\sqrt{\mu_o\varepsilon_1}$, $\qquad n_2 = c\sqrt{\mu_o\varepsilon_2}$,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{P_{t,ave}}{P_{i,ave}} = \frac{\eta_1}{\eta_2}\frac{E_{to}^{\ 2}}{E_{io}^{\ 2}} = \frac{\eta_1}{\eta_2}\tau^2 = \frac{\eta_1}{\eta_2}(1+\Gamma)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

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(b) If $P_{r,ave} = P_{t,ave} \longrightarrow RP_{i,ave} = TP_{i.ave} \longrightarrow R = T$

i.e. $(n_1 - n_2)^2 = 4n_1 n_2 \quad \longrightarrow \quad n_1^2 - 6n_1 n_2 + n_2^2 = 0$

or $\left(\frac{n_1}{n_2}\right)^2 - 6\left(\frac{n_1}{n_2}\right) + 1 = 0,$ so

$$\frac{n_1}{n_2} = 3 \pm \sqrt{8} = \underline{\underline{5.828 \quad \text{or} \quad 0.1716}}$$

(Note that these values are mutual reciprocals, reflecting the inherent symmetry of the problem.)

**Prob. 10.53** (a) $\eta_1 = \eta_o, \quad \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \eta_o / 2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o / 2 - \eta_o}{3\eta_o / 2} = \underline{\underline{-1/3}}, \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{\eta_o}{3\eta_o / 2} = \underline{\underline{2/3}}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3} = \underline{\underline{2}}$$

(b) $E_{or} = \Gamma E_{oi} = -\frac{1}{3} \times (30) = -10$

$$\boldsymbol{E}_r = -10\cos(\omega t + z)\boldsymbol{a}_x \text{ V/m}$$

Let $\boldsymbol{H}_r = H_{or}\cos(\omega t + z)\boldsymbol{a}_H$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow -\boldsymbol{a}_x \times \boldsymbol{a}_H = -\boldsymbol{a}_z \longrightarrow \boldsymbol{a}_H = \boldsymbol{a}_y$$

$$\boldsymbol{H}_r = \frac{10}{120\pi}\cos(\omega t + z)\boldsymbol{a}_y = \underline{\underline{26.53\cos(\omega t + z)\boldsymbol{a}_y}} \text{ mA/m}$$

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**Prob. 10.54**

$$\beta_1 = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3\times 10^8} = \frac{2\pi}{3}$$

$$\eta_1 = \eta_o, \quad \eta_2 = 0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \eta_o}{0 + \eta_o} = -1, \quad \tau = 1 + \Gamma = 0$$

$$\boldsymbol{E}_t = \boldsymbol{0}$$

$$\boldsymbol{E}_r = \underline{\underline{-50\sin(2\pi \times 10^8 t + \beta_1 x)\boldsymbol{a}_z \text{ V/m}}}$$

**Prob. 10.55** (a) Medium 1 is free space. Given that $\beta = 1$,

$$\beta = 1 = \frac{\omega}{c} \quad \longrightarrow \quad \omega = c = \underline{\underline{3\times 10^8 \text{ rad/s}}}$$

(b) $\eta_1 = \eta_o, \qquad \eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_o\sqrt{\frac{3}{12}} = \eta_o / 2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3, \qquad \tau = 1 + \Gamma = 2/3$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3} = \underline{\underline{2}}$$

(c) Let $\boldsymbol{H}_r = H_{or}\cos(\omega t + z)\boldsymbol{a}_H$, where

$$\boldsymbol{E}_r = -\frac{1}{3}(30)\cos(\omega t + z)\boldsymbol{a}_y = -10\cos(\omega t + z)\boldsymbol{a}_y, \qquad H_{or} = \frac{10}{\eta_o} = \frac{10}{120\pi}$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow -\boldsymbol{a}_y \times \boldsymbol{a}_H = -\boldsymbol{a}_z \longrightarrow \boldsymbol{a}_H = -\boldsymbol{a}_x$$

$$\boldsymbol{H}_r = -\frac{10}{120\pi}\cos(3\times 10^8 t + z)\boldsymbol{a}_x \text{ A/m} = \underline{\underline{-26.53\cos(3\times 10^8 t + z)\boldsymbol{a}_x \text{ mA/m}}}$$

(d) $\boldsymbol{P}_t = \frac{E_{ot}^2}{2\eta_2}\boldsymbol{a}_z, \qquad E_{ot} = \tau E_{oi} = \frac{2}{3}(30) = 20, \quad \eta_2 = 60\pi$

$$\boldsymbol{P}_t = \frac{20^2}{120\pi}(\boldsymbol{a}_z) = \underline{\underline{1.061\boldsymbol{a}_z \text{ W/m}^2}}$$

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**Prob. 10.56** $\eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}} = \eta_o / 2, \qquad \eta_2 = \eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1/3, \qquad \tau = 1 + \Gamma = 4/3$$

$$E_{or} = \Gamma E_{io} = (1/3)(5) = 5/3, \qquad E_{ot} = \tau E_{io} = 20/3$$

$$\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{10^8}{3\times 10^8}\sqrt{4} = 2/3$$

(a) $\boldsymbol{E}_r = \dfrac{5}{3}\cos(10^8 t - 2y/3)\boldsymbol{a}_z$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{5\cos(10^8 t + \frac{2}{3}y)\boldsymbol{a}_z + \frac{5}{3}\cos(10^8 t - \frac{2}{3}y)\boldsymbol{a}_z}} \text{ V/m}$$

(b) $$\mathcal{P}_{\text{ave1}} = \frac{E_{io}^{\,2}}{2\eta_1}(-\boldsymbol{a}_y) + \frac{E_{ro}^{\,2}}{2\eta_1}(+\boldsymbol{a}_y) = \frac{25}{2(60\pi)}(1 - \frac{1}{9})(-\boldsymbol{a}_y) = \underline{\underline{-0.0589\boldsymbol{a}_y \text{ W/m}^2}}$$

(c) $$\mathcal{P}_{\text{ave2}} = \frac{E_{to}^{\,2}}{2\eta_2}(-\boldsymbol{a}_y) = \frac{400}{9(2)(120\pi)}(-\boldsymbol{a}_y) = \underline{\underline{-0.0589\boldsymbol{a}_y \text{ W/m}^2}}$$

**Prob. 10.57**

(a) $\eta_1 = \eta_o$

$$\boldsymbol{E}_i = E_{io}\sin(\omega t - 5x)\boldsymbol{a}_E$$

$$E_{io} = H_{io}\eta_o = 120\pi \times 4 = 480\pi$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_E \times \boldsymbol{a}_y = \boldsymbol{a}_x \longrightarrow \boldsymbol{a}_E = -\boldsymbol{a}_z$$

$$\boldsymbol{E}_i = -480\pi\sin(\omega t - 5x)\boldsymbol{a}_z$$

$$\eta_2 = \sqrt{\frac{\mu_o}{4\varepsilon_o}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -1/3, \qquad \tau = 1 + \Gamma = 2/3$$

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$$E_{ro} = \Gamma E_{io} = (-1/3)(-480\pi) = 160\pi$$

$$\boldsymbol{E}_r = 160\pi \sin(\omega t + 5x)\boldsymbol{a}_z$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{-1.508\sin(\omega t - 5x)\boldsymbol{a}_z + 0.503\sin(\omega t + 5x)\boldsymbol{a}_z}} \text{ kV/m}$$

(b) $E_{to} = \tau E_{io} = (2/3)(480\pi) = 320\pi$

$$\mathcal{P} = \frac{E_{to}^{\ 2}}{2\eta_2}\boldsymbol{a}_x = \frac{(320\pi)^2}{2(60\pi)}\boldsymbol{a}_x = \underline{\underline{2.68\boldsymbol{a}_x}} \text{ kW/m}^2$$

(c) $s = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+1/3}{1-1/3} = \underline{\underline{2}}$

**Prob. 10.58**

(a) In air, $\beta_1 = 1, \lambda_1 = 2\pi/\beta_1 = 2\pi = \underline{\underline{6.283 \text{ m}}}$

$$\omega = \beta_1 c = \underline{\underline{3\times10^8 \text{ rad/s}}}$$

In the dielectric medium, $\omega$ is the same .

$$\omega = \underline{\underline{3\times10^8 \text{ rad/s}}}$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r2}} = \beta_1\sqrt{\varepsilon_{r2}} = \sqrt{3}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\sqrt{3}} = \underline{\underline{3.6276 \text{ m}}}$$

(b) $H_o = \dfrac{E_o}{\eta_o} = \dfrac{10}{120\pi} = 0.0265$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_z \times \boldsymbol{a}_y = -\boldsymbol{a}_x$$

$$\boldsymbol{H}_i = \underline{\underline{-26.5\cos(\omega t - z)\boldsymbol{a}_x}} \text{ mA/m}$$

(c) $\eta_1 = \eta_o, \quad \eta_2 = \eta_o/\sqrt{3}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1/\sqrt{3}) - 1}{(1/\sqrt{3}) + 1} = \underline{\underline{-0.268}}, \qquad \tau = 1 + \Gamma = \underline{\underline{0.732}}$$

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(d) $E_{to} = \tau E_{io} = 7.32, \quad E_{ro} = \Gamma E_{io} = -2.68$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{10\cos(\omega t - z)\boldsymbol{a}_y - 2.68\cos(\omega t + z)\boldsymbol{a}_y \text{ V/m}}}$$

$$\boldsymbol{E}_2 = \boldsymbol{E}_t = \underline{\underline{7.32\cos(\omega t - z)\boldsymbol{a}_y \text{ V/m}}}$$

$$\mathcal{P}_{\text{ave1}} = \frac{1}{2\eta_1}(\boldsymbol{a}_z)[E_{io}^{\,2} - E_{ro}^{\,2}] = \frac{1}{2(120\pi)}(\boldsymbol{a}_z)(10^2 - 2.68^2) = \underline{\underline{0.1231\boldsymbol{a}_z \text{ W/m}^2}}$$

$$\mathcal{P}_{\text{ave2}} = \frac{E_{to}^{\,2}}{2\eta_2}(\boldsymbol{a}_z) = \frac{\sqrt{3}}{2\times 120\pi}(7.32)^2(\boldsymbol{a}_z) = \underline{\underline{0.1231\boldsymbol{a}_z \text{ W/m}^2}}$$

**Prob. 10.59**

$\eta_1 = \eta_o = 120\pi$

For seawater (lossy medium),

$$\eta_2 = \sqrt{\frac{j\omega\mu_o}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j2\pi\times 10^8 \times 4}{4 + j2\pi\times 10^8 \times 81 \times \frac{10^{-9}}{36\pi}}} = 10.44 + j9.333$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.9461\angle 177.16$$

$|\Gamma|^2 = 0.8952, \quad 1 - |\Gamma| = 0.1084$

$$\frac{P_r}{P_i} = \underline{\underline{89.51\%}}, \quad \frac{P_t}{P_i} = \underline{\underline{10.84\%}},$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{7.924\angle 43.975 - 377}{7.924\angle 43.975 + 377} = 0.9702\angle 178.2^o$$

The fraction of the incident power reflected is

$$\frac{P_r}{P_i} = |\Gamma|^2 = 0.9702^2 = \underline{\underline{0.9413}}$$

The transmitted fraction is

$$\frac{P_t}{P_i} = 1 - |\Gamma|^2 = 1 - 0.9702^2 = \underline{\underline{0.0587}}$$

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**Prob. 10.60**

(a)

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \frac{120\pi}{\sqrt{4}} = 188.5, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{3.2}} = 210.75$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{210.75 - 188.5}{210.75 + 188.5} = 0.0557, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 210.75}{210.75 + 188.5} = 1.0557$$

$$E_{ro} = \Gamma E_{io} = (0.0557)(12) = 0.6684 \qquad E_{to} = \tau E_{io} = 1.0557(12) = 12.668$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = \frac{\omega}{c}\sqrt{4} \quad \longrightarrow \quad \omega = \frac{\beta_1 c}{2} = \frac{40\pi(3\times10^8)}{2} = \underline{\underline{6\pi \times 10^9 \text{ rad/s}}}$$

(b)

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \frac{\omega}{c}\sqrt{3.2} = \frac{6\pi\times10^9\sqrt{3.2}}{3\times10^8} = 112.4$$

$$\boldsymbol{E}_r = E_{ro}\cos(\omega t + 40\pi x)\boldsymbol{a}_z = \underline{\underline{0.6684\cos(6\pi\times10^9 t + 40\pi x)\boldsymbol{a}_z \text{ V/m}}}$$

$$\boldsymbol{E}_t = E_{to}\cos(\omega t - \beta_2 x)\boldsymbol{a}_z = \underline{\underline{12.668\cos(6\pi\times10^9 t - 112.4x)\boldsymbol{a}_z \text{ V/m}}}$$

**Prob. 10.61** (a) $\omega = \beta c = 3\times3\times10^8 = \underline{\underline{9\times10^8 \text{ rad/s}}}$

(b) $\lambda = 2\pi / \beta = 2\pi / 3 = \underline{\underline{2.094 \text{ m}}}$

(c) $$\frac{\sigma}{\omega\varepsilon} = \frac{4}{9\times10^8 \times 80\times10^{-9} / 36\pi} = 2\pi = \underline{\underline{6.288}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 6.288 \quad \longrightarrow \quad \theta_\eta = 40.47^o$$

$$|\eta_2| = \frac{\sqrt{\mu_2 / \varepsilon_2}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{377/\sqrt{80}}{\sqrt[4]{1+4\pi^2}} = 16.71$$

$$\eta_2 = \underline{\underline{16.71\angle 40.47^o \ \Omega}}$$

(d) $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{16.71\angle 40.47^o - 377}{16.71\angle 40.47^o + 377} = 0.935\angle 179.7^o$$

$$E_{or} = \Gamma E_{oi} = 9.35\angle 179.7^o$$

$$\boldsymbol{E}_r = \underline{\underline{9.35\sin(\omega t - 3z + 179.7)\boldsymbol{a}_x \text{ V/m}}}$$

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$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{\mu_{r2}\varepsilon_{r2}}{2}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}-1\right]} = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2}-1\right]} = 43.94 \text{ Np/m}$$

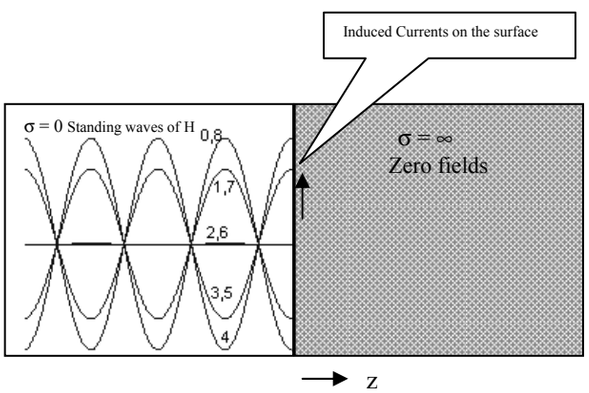
$$\beta_2 = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2}+1\right]} = 51.48 \text{ rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{2\times16.71\angle40.47^o}{16.71\angle40.47^o+377} = 0.0857\angle38.89^o$$

$$E_{ot} = \tau E_o = 0.857\angle38.89^o$$

$$\boldsymbol{E}_t = 0.857e^{43.94z}\sin(9\times10^8 t + 51.48z + 38.89^o)\boldsymbol{a}_x \text{ V/m}$$

**Prob. 10.62**




Curve 0 is at t = 0; curve 1 is at t = T/8; curve 2 is at t = T/4; curve 3 is at t = 3T/8, etc.

**Prob. 10.63** Since $\mu_o = \mu_1 = \mu_2$,

$$\sin\theta_{t1} = \sin\theta_i\sqrt{\frac{\varepsilon_o}{\varepsilon_1}} = \frac{\sin45^o}{\sqrt{4.5}} = 0.3333 \quad \longrightarrow \quad \underline{\underline{\theta_{t1} = 19.47^o}}$$

$$\sin\theta_{t2} = \sin\theta_{t1}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{1}{3}\sqrt{\frac{4.5}{2.25}} = 0.4714 \quad \longrightarrow \quad \underline{\underline{\theta_{t2} = 28.13^o}}$$

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**Prob. 10.64**

$$\boldsymbol{E}_s = \frac{20(e^{jk_x x} - e^{-jk_x x})}{j2}\frac{(e^{jk_y y} + e^{-jk_y y})}{2}\boldsymbol{a}_z$$

$$= -j5\left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)}\right]\boldsymbol{a}_z$$

which consists of four plane waves.

$$\nabla \times \boldsymbol{E}_s = -j\omega\mu_o \boldsymbol{H}_s \quad \longrightarrow \quad \boldsymbol{H}_s = \frac{j}{\omega\mu_o}\nabla \times \boldsymbol{E}_s = \frac{j}{\omega\mu_o}\left(\frac{\partial E_z}{\partial y}\boldsymbol{a}_x - \frac{\partial E_z}{\partial x}\boldsymbol{a}_y\right)$$

$$\boldsymbol{H}_s = -\frac{j20}{\omega\mu_o}\left[k_y \sin(k_x x)\sin(k_y y)\boldsymbol{a}_x + k_x \cos(k_x x)\cos(k_y y)\boldsymbol{a}_y\right]$$

**Prob. 10.65**

$\eta_1 = \eta_o = 377\Omega$

For $\eta_2$,

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{4}{2\pi \times 1.2 \times 10^9 \times 50 \times \frac{10^{-9}}{36\pi}} = 1.2$$

$$\tan 2\theta_{\eta_2} = \frac{\sigma_2}{\omega\varepsilon_2} = 1.2 \quad \longrightarrow \quad \theta_{\eta_2} = 25.1^o$$

$$|\eta_2| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{120\pi\sqrt{1/50}}{\sqrt[4]{1 + 1.2^2}} = 42.658$$

$$\eta_2 = 42.658\angle 25.1^o$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{42.658\angle 25.1^o - 377}{42.658\angle 25.1^o + 377} = \underline{\underline{0.8146\angle 174.4^o}}$$

**Prob. 10.66**

(a)

$$P_t = (1 - |\Gamma|^2)P_i$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \longrightarrow \quad |\Gamma| = \frac{s-1}{s+1}$$

$$\frac{P_t}{P_i} = 1 - \left(\frac{s-1}{s+1}\right)^2 = \underline{\underline{\frac{4s}{(s+1)^2}}}$$

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(b) $P_i = P_r + P_t \quad \longrightarrow \quad \frac{P_r}{P_i} = 1 - \frac{P_t}{P_i} = \underline{\underline{\left(\frac{s-1}{s+1}\right)^2}}$

**Prob. 10.67**

If $\mathbf{A}$ is a uniform vector and $\Phi(r)$ is a scalar,

$$\nabla \times (\Phi \boldsymbol{A}) = \nabla\Phi \times \boldsymbol{A} + \Phi(\nabla \times \boldsymbol{A}) = \nabla\Phi \times \boldsymbol{A}$$

since $\nabla \times \boldsymbol{A} = \boldsymbol{0}$.

$$\nabla \times \boldsymbol{E} = (\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\mathrm{a}_z) \times \boldsymbol{E}_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x \boldsymbol{a}_x + k_y \boldsymbol{a}_y + k_z \boldsymbol{a}_z) e^{j(\boldsymbol{k} \bullet \boldsymbol{r} - \omega t)} \times \boldsymbol{E}_o$$

$$= j\boldsymbol{k} \times \boldsymbol{E}_o e^{j(\boldsymbol{k} \bullet \boldsymbol{r} - \omega t)} = j\boldsymbol{k} \times \boldsymbol{E}$$

Also, $-\frac{\partial \boldsymbol{B}}{\partial t} = j\omega\mu\boldsymbol{H}$. Hence $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ becomes $\boldsymbol{k} \times \boldsymbol{E} = \omega\mu\boldsymbol{H}$

From this, $\boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_H$

**Prob. 10.68**

$$k = |\mathbf{k}| = \sqrt{124^2 + 124^2 + 263^2} = 316.1$$

$$\lambda = \frac{2\pi}{k} = \underline{\underline{19.88 \text{ mm}}}$$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} \quad \longrightarrow \quad f = \frac{kc}{2\pi} = \frac{316.1 \times 3 \times 10^8}{2\pi} = \underline{\underline{15.093 \text{ GHz}}}$$

$$\boldsymbol{k} \bullet \boldsymbol{a}_x = k\cos\theta_x \quad \longrightarrow \quad \cos\theta_x = \frac{124}{316.1} \quad \longrightarrow \quad \theta_x = 66.9^o = \theta_y$$

$$\theta_z = \cos^{-1}\frac{263}{361.1} = 33.69^o$$

Thus,

$$\underline{\underline{\theta_x = \theta_y = 66.9^o, \ \theta_z = 33.69^o}}$$

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**Prob. 10.69**

$$\boldsymbol{k} = -3.4\boldsymbol{a}_x + 4.2\boldsymbol{a}_y$$

$$\boldsymbol{k}\bullet\boldsymbol{E} = 0 \quad \longrightarrow \quad 0 = -3.4E_o + 4.2$$

$$E_o = \frac{4.2}{3.4} = \underline{\underline{1.235}}$$

$$k = |\,\boldsymbol{k}\,| = \beta = \sqrt{(-3.4)^2 + (4.2)^2} = 5.403$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.403} = 1.162$$

$$f = \frac{c}{\lambda} = \frac{3\times 10^8}{1.162} = \underline{\underline{258 \text{ MHz}}}$$

$$\boldsymbol{H}_s = \frac{1}{\mu\omega}\boldsymbol{k}\times\boldsymbol{E}_s = \frac{1}{\mu kc}\boldsymbol{k}\times\boldsymbol{E}_s$$

$$= \frac{1}{4\pi\times 10^{-7}\times 5.403\times 3\times 10^8}\begin{vmatrix} -3.4 & 4.2 & 0 \\ E_o & 1 & 3+j4 \end{vmatrix} A_o$$

where $A_o = e^{-j3.4x+4.2y}$

$$\boldsymbol{H}_s = 4.91A_o\times 10^{-4}\left[4.2(3+j4)\boldsymbol{a}_x + 3.4(3+j4)\boldsymbol{a}_y + (-3.4-4.2E_o)\boldsymbol{a}_z\right]$$

$$= 0.491\left[(12.6+j16.8)\boldsymbol{a}_x + (10.2+j13.6)\boldsymbol{a}_y - 8.59\boldsymbol{a}_z\right]e^{-j3.4x+4.2y} \text{ mA/m}$$

**Prob.10.70**

$$\nabla\bullet\boldsymbol{E} = \left(\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\boldsymbol{a}_z\right)\bullet\boldsymbol{E}_o e^{j(k_x x+k_y y+k_z z-\omega t)} = j(k_x\boldsymbol{a}_x + k_y\boldsymbol{a}_y + k_z\boldsymbol{a}_z)e^{j(\boldsymbol{k}\bullet\boldsymbol{r}-\omega t)}\bullet\boldsymbol{E}_o$$

$$= j\boldsymbol{k}\bullet\boldsymbol{E}_o e^{j(\boldsymbol{k}\bullet\boldsymbol{r}-\omega t)} = j\boldsymbol{k}\bullet\boldsymbol{E} = 0 \quad \longrightarrow \quad \boldsymbol{k}\bullet\boldsymbol{E} = 0$$

Similarly,

$$\nabla\bullet\boldsymbol{H} = j\boldsymbol{k}\bullet\boldsymbol{H} = 0 \quad \longrightarrow \quad \boldsymbol{k}\bullet\boldsymbol{H} = 0$$

It has been shown in the previous problem that

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \quad \longrightarrow \quad \boldsymbol{k}\times\boldsymbol{E} = \omega\mu\boldsymbol{H}$$

Similarly,

$$\nabla\times\boldsymbol{H} = \frac{\partial\boldsymbol{D}}{\partial t} \quad \longrightarrow \quad kxH = -\varepsilon\omega E$$

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From $\boldsymbol{k}\times\boldsymbol{E}=\omega\mu\boldsymbol{H}$, $\quad \boldsymbol{a}_k\times\boldsymbol{a}_E=\boldsymbol{a}_H$ and

From $\boldsymbol{k}\times\boldsymbol{H}=-\varepsilon\omega\boldsymbol{E}$, $\quad \boldsymbol{a}_k\times\boldsymbol{a}_H=-\boldsymbol{a}_E$

**Prob. 10.71**

If $\mu_o=\mu_1=\mu_2,\quad \eta_1=\dfrac{\eta_o}{\sqrt{\varepsilon_{r1}}},\eta_2=\dfrac{\eta_o}{\sqrt{\varepsilon_{r2}}}$

$$\Gamma_{\backslash\backslash}=\frac{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t-\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t+\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}$$

$$\sqrt{\varepsilon_{r1}}\sin\theta_i=\sqrt{\varepsilon_{r2}}\sin\theta_t \quad\longrightarrow\quad \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}}=\frac{\sin\theta_i}{\sin\theta_t}$$

$$\Gamma_{\backslash\backslash}=\frac{\cos\theta_t-\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}{\cos\theta_t+\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}=\frac{\sin\theta_t\cos\theta_t-\sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t+\sin\theta_i\cos\theta_i}$$

$$=\frac{\sin 2\theta_t-\sin 2\theta_i}{\sin 2\theta_t+\sin 2\theta_i}=\frac{\sin(\theta_t-\theta_i)\cos(\theta_t+\theta_i)}{\cos(\theta_t-\theta_i)\sin(\theta_t+\theta_i)}=\frac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}$$

Similarly,

$$\tau_{\backslash\backslash}=\frac{\dfrac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t+\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}=\frac{2\cos\theta_i}{\cos\theta_t+\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{\sin\theta_t\cos\theta_t(\sin^2\theta_i+\cos^2\theta_i)+\sin\theta_i\cos\theta_i(\sin^2\theta_t+\cos^2\theta_t)}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{(\sin\theta_i\cos\theta_t+\sin\ \theta_t\cos\ \theta_i)(\cos\theta_i\cos\theta_t+\sin\ \theta_i\sin\ \theta_t)}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

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$$\Gamma_\perp = \frac{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{\cos\theta_i - \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\tau_\perp = \frac{\frac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{2\cos\theta_i \sin\theta_i}{\sin(\theta_t + \theta_i)}$$

**Prob. 10.72**

(a) $n_1 = 1, \quad n_2 = c\sqrt{\mu_2\varepsilon_2} = c\sqrt{6.4\varepsilon_o \times \mu_o} = \sqrt{6.4} = 2.5298$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{1}{2.5298}\sin 12^o = 0.082185 \quad \longrightarrow \quad \theta_t = 4.714^o$$

$$\eta_1 = 120\pi, \qquad \eta_2 = 120\pi\sqrt{\frac{1}{6.4}} = 47.43\pi$$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{47.43\pi\cos 4.714^o - 120\pi\cos 12^o}{47.43\pi\cos 4.714^o + 120\pi\cos 12^o}$$

$$= \frac{47.27 - 117.38}{47.27 + 117.38} = \underline{\underline{-0.4258}}$$

$$\frac{E_{to}}{E_{io}} = \tau = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{2x47.43\cos 12^o}{47.27 + 117.33} = \frac{92.787}{164.65} = \underline{\underline{0.5635}}$$

**Prob. 10.73**

(a) $\boldsymbol{k}_i = 4\boldsymbol{a}_y + \mathbf{3}a_z$

$$\boldsymbol{k}_i \bullet \boldsymbol{a}_n = k_i\cos\theta_i \quad \longrightarrow \quad \cos\theta_i = 4/5 \quad \longrightarrow \quad \underline{\underline{\theta_i = 36.87^o}}$$

(b)

$$\boldsymbol{P}_{\text{ave}} = \frac{1}{2}\,\text{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s^*) = \frac{E_o^{\,2}}{2\eta}\boldsymbol{a}_k = \frac{(\sqrt{8^2 + 6^2})^2}{2\times 120\pi}\frac{(4\boldsymbol{a}_y + 3\boldsymbol{a}_z)}{5} = \underline{\underline{106.1\boldsymbol{a}_y + 79.58\boldsymbol{a}_z \text{ mW/m}^2}}$$

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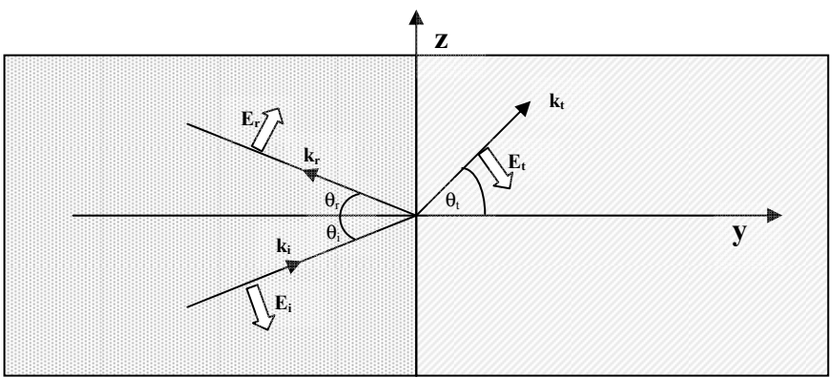
(c) $\theta_r = \theta_i = 36.87^o$. Let

$$\boldsymbol{E}_r = (E_{ry}\boldsymbol{a}_x + E_{rz}\boldsymbol{a}_z)\sin(\omega t - \boldsymbol{k}_r \bullet \boldsymbol{r})$$




From the figure, $\boldsymbol{k}_r = k_{rz}\boldsymbol{a}_z - k_{ry}\boldsymbol{a}_y$. But $k_r = k_i = 5$

$$k_{rz} = k_r \sin\theta_r = 5(3/5) = 3, \quad k_{ry} = k_r \cos\theta_r = 5(4/5) = 4,$$

Hence, $\boldsymbol{k}_r = -4\boldsymbol{a}_y + 3\boldsymbol{a}_z$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{3/5}{\sqrt{4}} = 0.3$$

$$\theta_t = 17.46, \quad \cos\theta_t = 0.9539, \quad \eta_1 = \eta_o = 120\pi, \eta_2 = \eta_o/2 = 60\pi$$

$$\Gamma_{//} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\frac{\eta_o}{2}(0.9539) - \eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = -0.253$$

$$E_{ro} = \Gamma_{//}E_{io} = -0.253(10) = -2.53$$

But $(E_{ry}\boldsymbol{a}_y + E_{rz}\boldsymbol{a}_z) = E_{ro}(\sin\theta_r\boldsymbol{a}_y + \cos\theta_r\boldsymbol{a}_z) = -2.53(\frac{3}{5}\boldsymbol{a}_y + \frac{4}{5}\boldsymbol{a}_z)$

$$\boldsymbol{E}_r = -(1.518\boldsymbol{a}_y + 2.024\boldsymbol{a}_z)\sin(\omega t + 4y - 3z) \text{ V/m}$$

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Similarly, let

$$\boldsymbol{E}_t = (E_{ty}\boldsymbol{a}_y + E_{tz}\boldsymbol{a}_z)\sin(\omega t - \boldsymbol{k}_t \bullet \boldsymbol{r})$$

$$k_t = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{4\mu_o\varepsilon_o}$$

But $k_i = \beta_1 = \omega\sqrt{\mu_o\varepsilon_o}$

$$\frac{k_t}{k_i} = 2 \quad \longrightarrow \quad k_t = 2k_i = 10$$

$$k_{ty} = k_t\cos\theta_t = 9.539, \quad k_{tz} = k_t\sin\theta_t = 3,$$

$$k_t = 9.539a_y + 3a_z$$

Note that $k_{iz} = k_{rz} = k_{tz} = 3$

$$\tau_{\shortparallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = 0.6265$$

$$E_{to} = \tau_{\shortparallel}E_{io} = 6.265$$

But

$$(E_{ty}\boldsymbol{a}_y + E_{tz}\boldsymbol{a}_z) = E_{to}(\sin\theta_t\boldsymbol{a}_y - \cos\theta_t\boldsymbol{a}_z) = 6.256(0.3\boldsymbol{a}_y - 0.9539\boldsymbol{a}_z)$$

Hence,

$$\underline{\underline{\boldsymbol{E}_t = (1.879\boldsymbol{a}_y - 5.968\boldsymbol{a}_z)\sin(\omega t - 9.539y - 3z)}} \text{ V/m}$$

**Prob. 10.74**

(a) $n = \dfrac{c}{u} = \sqrt{\mu_r\varepsilon_r} = \sqrt{2.1\times 1} = \underline{\underline{1.45}}$

(b) $n = \sqrt{\mu_r\varepsilon_r} = \sqrt{1\times 81} = \underline{\underline{9}}$

(c) $n = \sqrt{\varepsilon_r} = \sqrt{2.7} = \underline{\underline{1.643}}$

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**Prob. 10.75**

(a) From air to seawater,

$$\varepsilon_{r1} = 1, \quad \varepsilon_{r2} = 81$$

$$\tan\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{81}{1}} = 9 \quad \longrightarrow \quad \underline{\underline{\theta_B = 83.66^o}}$$

(b) From seawater to air,

$$\varepsilon_{r1} = 81, \quad \varepsilon_{r2} = 1$$

$$\tan\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{1}{81}} = \frac{1}{9} \quad \longrightarrow \quad \underline{\underline{\theta_B = 6.34^o}}$$

**Prob. 10.76**

(a)

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{1}{\sqrt{8}} \quad \longrightarrow \quad \underline{\underline{\theta_i = \theta_r = 19.47^o}}$$

$$\sin\theta_t = \sin\theta_i \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}(3) = 1 \quad \longrightarrow \quad \underline{\underline{\theta_t = 90^o}}$$

(b) $$\beta_1 = \frac{\omega}{c}\sqrt{\varepsilon_{r1}} = \frac{10^9}{3\times 10^8}\times 3 = 10 = k\sqrt{1+8} = 3k \quad \longrightarrow \quad \underline{\underline{k = 3.333}}$$

(c) $$\lambda = 2\pi/\beta, \quad \lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283}}\ \text{m}$$

$$\beta_2 = \omega/c = 10/3, \quad \lambda_2 = 2\pi/\beta_2 = 2\pi\times 3/10 = \underline{\underline{1.885}}\ \text{m}$$

(d)
$$\boldsymbol{E}_i = \eta_1 \boldsymbol{H}_x \times \boldsymbol{a}_k = 40\pi(0.2)\cos(\omega t - \boldsymbol{k}\bullet\boldsymbol{r})\boldsymbol{a}_y \times \frac{(\boldsymbol{a}_x + \sqrt{8}\boldsymbol{a}_z)}{3}$$

$$= \underline{\underline{(23.6954\boldsymbol{a}_x - 8.3776\boldsymbol{a}_z)\cos(10^9 t - kx - k\sqrt{8}z)\ \text{V/m}}}$$

(e) $$\tau_{//} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i+\theta_t)\cos(\theta_t-\theta_i)} = \frac{2\cos 19.47^o \sin 90^o}{\sin 19.47^o \cos 19.47^o} = 6$$

$$\Gamma_{//} = -\frac{\cot 19.47^o}{\cot 19.47^o} = -1$$

Let $$\boldsymbol{E}_t = -E_{io}(\cos\theta_t \boldsymbol{a}_x - \sin\theta_t \boldsymbol{a}_z)\cos(10^9 t - \beta_2 x\sin\theta_t - \beta_2 z\cos\theta_t)$$

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where

$$\boldsymbol{E}_t = -E_{io}(\cos\theta_t \boldsymbol{a}_x - \sin\theta_t \boldsymbol{a}_z)\cos(10^9 t - \beta_1 x\sin\theta_i - \beta_1 z\cos\theta_i)$$

$$\sin\theta_t = 1, \quad \cos\theta_t = 0, \quad \beta_2 \sin\theta_t = 10/3$$

$$E_{to}\sin\theta_t = \tau_{\backslash\backslash} E_{io} = 6(24\pi)(3)(1) = 1357.2$$

Hence,

$$\boldsymbol{E}_t = 1357\cos(10^9 t - 3.333x)\boldsymbol{a}_z \text{ V/m}$$

Since $\Gamma = -1, \quad \theta_r = \theta_i$

$$\underline{\underline{\boldsymbol{E}_r = (213.3\boldsymbol{a}_x + 75.4\boldsymbol{a}_z)\cos(10^9 t - kx + k\sqrt{8}z)}} \text{ V/m}$$

(f) $$\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{\varepsilon_o}{9\varepsilon_o}} = 1/3 \longrightarrow \underline{\underline{\theta_{B//} = 18.43^o}}$$

**Prob.10.77**

Microwave is used:

(1) For surveying land with a piece of equipment called the *tellurometer.* This radar system can precisely measure the distance between two points.
(2) For guidance. The guidance of missiles, the launching and homing guidance of space vehicles, and the control of ships are performed with the aid of microwaves.
(3) In semiconductor devices. A large number of new microwave semiconductor devices have been developed for the purpose of microwave oscillator, amplification, mixing/detection, frequency multiplication, and switching. Without such achievement, the majority of today's microwave systems could not exist.

**Prob.10.78**

(a) In terms of the S-parameters, the T-parameters are given by

$T_{11} = 1/S_{21}$, $T_{12} = -S_{22}/S_{21}$, $T_{21} = S_{11}/S_{21}$, $T_{22} = S_{12} - S_{11}S_{22}/S_{21}$

(b) $T_{11} = 1/0.4 = 2.5$, $T_{12} = -0.2/0.4$,

$T_{21} = 0.2/0.4$, $T_{22} = 0.4 - 0.2 \times 0.2/0.4 = 0.3$

Hence,

$$\mathrm{T} = \begin{bmatrix} 2.5 & -0.5 \\ 0.5 & 0.3 \end{bmatrix}$$

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**Prob. 10.79**

Since $Z_L = Z_o$, $\Gamma_L = 0$.

$$\Gamma_i = S_{11} = \underline{0.33 - j0.15}$$

$$\Gamma_g = (Z_g - Z_o)/(Z_g + Z_o) = (2 - 1)/(2 + 1) = 1/3$$

$$\Gamma o = S_{22} + S_{12}S_{21}\Gamma_g /(1 - S_{11}\Gamma_g)$$

$$= 0.44 - j0.62 + 0.56x0.56\ x(1/3)/[1 - (0.11 - j0.05)]$$

$$= \underline{0.5571 - j0.6266}$$

**Prob. 10.80** The microwave wavelengths are of the same magnitude as the circuit components. The wavelength in air at a microwave frequency of 300 GHz, for example, is 1 mm. The physical dimension of the lumped element must be in this range to avoid interference. Also, the leads connecting the lumped element probably have much more inductance and capacitance than is needed.

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